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Random Vibrations of Nonlinear Continua Endowed with Fractional Derivative Elements

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Abstract

In this paper, two techniques are proposed for determining the large displacement statistics of random exciting continua endowed with fractional derivative elements: Boundary Element Method (BEM) based Monte Carlo simulation; and Statistical Linearization (SL). The techniques are applied to the problem of nonlinear beam and plate random response determination in the case of colored random external load. The BEM is implemented in conjunction with a Newmark scheme for estimating the system response in the time domain in conjunction with repeated simulations, while SL is used for estimating efficiently and directly, albeit iteratively, the response statistics.

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Keywords: continuum mechanics; fractional derivative; boundary element method; statistical linearization.

1. Introduction

Fractional operators have received recently considerable attention in a number of thematic domains [1]. In this regard, a notable example pertains to viscoelasticity as discussed in the works, for instance, of Nutting [2], Gemant [3], Scott-Blair and Gaffyn [4], and Bagley and Torvik [5]. The impact of fractional operators in problems involving the vibration of systems endowed with fractional derivative elements was described in two extensive review articles by Rossikhin and Shitikova [6, 7] in a deterministic setting, while analyses focusing on systems exposed to random loads are more recent. In this context, Spanos and Zeldin [8] elucidated certain pitfalls associated with the use of frequency dependent parameters in conjunction with the calculation of system response statistics. Linear systems

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were investigated by Agrawal [9], and Di Paola, Failla, et al. [10], while nonlinear systems were investigated by techniques such as SL, stochastic averaging, Weiner path integrals and harmonic wavelet based SL [11-15].
The vibration problem of continuous systems has received some attention, as well. In this context, most articles deal with the analysis of either linear or nonlinear beams [16-19], and of plates [20-22]. Despite these efforts, the need of techniques for estimating the nonlinear response of continuous systems endowed with fractional derivative elements reliably and efficiently persists. Thus, this paper describes a solution technique based on the SL technique that allows estimating efficiently the response statistics of continuous systems and, further, a strictly numerical approach based on the BEM, for conducting Monte Carlo studies. The techniques are applied to the specific problems of large beam and plate displacements.

2. Governing equations

2.1. Preliminary concepts on fractional derivatives

A critical concept underlining the definition of fractional derivative relates to the definition of fractional integral, which is obtained as the convolution of a function $w(t)$ with a power law kernel. That is,

$$ {}_0D_t^{-\alpha} = \frac{1}{\Gamma(\alpha)}\int_0^t \frac{w(\tau)}{(t-\tau)^{-\alpha+1}}d\tau \text{ , for } \alpha > 0, \tag{1} $$

with $\Gamma(\alpha)$ being the gamma function [23]. Clearly, for integer values of the power law, $\alpha = n$, the gamma function renders the factorial of the integer number and, thus eq. (1) provides the classical n-fold integral. The Riemann-Liouville (RL) fractional derivative is constructed by differentiating eq. (1) m times. That is,

$$ {}_0^{RL}D_t^{\alpha} = \frac{1}{\Gamma(m-\alpha)}\frac{d^m}{dt^m}\int_0^t \frac{w(\tau)}{(t-\tau)^{\alpha+1-m}}d\tau \text{ , } m\text{-}1 \le \alpha < m. \tag{2} $$

Further, the Grünwald-Letnikov (GL) representation [24] is given by the equation

$$ {}_0^{GL}D_t^{\alpha}w(t) = \sum_{k=0}^{m-1}\frac{w^{(k)}(0)t^{k-\alpha}}{\Gamma(k+1-\alpha)} + \frac{1}{\Gamma(m-\alpha)}\int_0^t \frac{w^{(m)}(\tau)}{(t-\tau)^{\alpha+1-m}}d\tau \text{ , } m\text{-}1 < \alpha < m. \tag{3} $$

Such a representation can lead to algorithms for the numerical computation of fractional derivatives. Indeed, the series in eq. (3) can be expanded and the series representation of the GL derivative

$$ {}_0^{GL}D_t^{\alpha}w(t) = \lim_{\Delta t \to 0}\Delta t^{-\gamma}\sum_{k=0}^{n}GL_k w(t-k\Delta t) \text{ ,} \tag{4} $$

can be derived, where GL_k are calculated recursively using the relationship

$$ GL_k = \frac{k-\alpha-1}{k}GL_{k-1}\text{; } GL_0 = 1 . \tag{5} $$

Eq. (4) provides the G1-algorithm that is used in this paper for conducting requisite numerical integrations. Such an algorithm clearly points out the fading memory property of the fractional derivative.