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# Acoustic Black Holes for Flexural Waves: A Smart Approach to Vibration Damping

Victor V. Krylov*

*Department of Aeronautical and Automotive Engineering, Loughborough University, Loughborough, Leicestershire, LE11 3TU, UK*

**Abstract**

The present paper provides a brief review of the theoretical and experimental investigations of 'acoustic black holes' for flexural waves in plate-like structures. Such acoustic black holes are relatively new physical objects that can absorb almost 100% of the incident wave energy. This makes them attractive for vibration damping in plate-like structures. The main principle of the acoustic black holes is based on a linear or higher order decrease in velocity of the incident flexural wave with propagation distance to almost zero. The decrease in velocity should be accompanied by efficient energy absorption in the area of very low velocity via insertion of small pieces of absorbing materials. This principle can be applied to achieve efficient damping of flexural waves and vibrations in plate-like structures using both one-dimensional acoustic black holes (power-law-profiled wedges) and two-dimensional acoustic black holes (power-law-profiled cylindrical indentations). The key advantage of using acoustic black holes for damping structural vibrations is that it requires very small amounts of added damping materials, in comparison with traditional methods, which is especially important for vibration damping in light-weight structures.


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*Keywords:* Acoustic black holes; Flexural waves; Vibration damping.

## 1. Introduction

Damping of structural vibrations is an important engineering problem in many areas, especially in transport engineering. The classical solution to this problem for metallic structures is to cover all their surfaces by layers of

* Corresponding author. Tel.: +44-01509-227216; fax: +44-01509-227275.
*E-mail address:* v.v.krylov@lboro.ac.uk

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absorbing materials, usually made of polymers or rubber [1,2]. This method of vibration damping involves adding of additional mass to structures, which is costly and not always acceptable. As an alternative to this, a new method of structural vibration damping using 'acoustic black holes' for flexural waves has been developed and investigated [3-9]. Acoustic black holes for flexural waves in plate-like structures can absorb almost 100% of the incident wave energy, which makes them attractive for damping structural vibrations. The main principle of acoustic black holes is based on a linear or higher order power-law-type decrease in velocity of the incident wave with propagation distance to almost zero accompanied by efficient energy absorption in the area of low velocity via small pieces of inserted absorbing materials. In the case of flexural waves in plates, the required gradual reduction in wave velocity with distance can be easily achieved by changing the plate local thickness according to a power law, with the power-law exponent being equal or larger than two. This smart approach to structural vibration damping has been applied to achieve efficient damping of flexural vibrations in plate-like structures using both one-dimensional ‘acoustic black holes’ (power-law wedges with their sharp edges covered by narrow strips of absorbing materials) and two-dimensional ‘acoustic black holes’ (power-law-profiled pits with small pieces of absorbing materials attached in the middle). The key advantage of using the above-mentioned acoustic black holes for damping structural vibrations is that they require very small amounts of added damping materials, in comparison with traditional methods, which is especially important for vibration damping in light-weight structures.

The present paper provides a brief review of the theory of acoustic black holes and of the recent theoretical experimental investigations carried out at Loughborough University and in other parts of the world on damping structural vibrations using acoustic black holes.

## 2. Outline of the Theory

### *2.1 One-dimensional acoustic black holes*

The principle of acoustic black holes can be explained by considering a general case of one-dimensional wave propagation characterised by the distance $x$ in an ideal medium with power-law dependence of wave velocity $c$ on $x$ as $c = ax^n$, where $n$ is a positive rational number and $a$ is a constant. One can express the geometrical acoustics solution for the complex amplitude $U(x)$ of a wave propagating from any arbitrary point $x$ towards a zero point (where $c = 0$) as

$$U(x) = A(x)e^{i\Phi(x)} . \tag{1}$$

Here

$$\Phi = -\int_x^0 k(x)dx = \int_0^x k(x)dx \tag{2}$$

is a total accumulated phase, and $A(x)$ is a slowly varying amplitude. Since $k(x) = \omega/c(x) = \omega/ax^n$, it is obvious from Eqn (2) that for $n \geq 1$ the integrals in (2) diverge and the phase $\Phi$ becomes infinite. This means that under these circumstances the wave never reaches the edge. Therefore, it never reflects back either, i.e. the wave becomes trapped, thus indicating that the above mentioned ideal medium with a linear or higher power-law profile of wave velocity can be considered as ‘acoustic black hole’ for the wave under consideration.

This phenomenon of zero reflection has been first described by Pekeris [10] in 1946 for sound waves in a stratified ocean with sound velocity profile linearly decreasing to zero. Later on, other authors have considered the possibility of the effects of zero reflection for waves of different physical nature, e.g. for internal water waves in a horizontally inhomogeneous stratified fluid [11]. Mironov [12] was the first to predict a practically important possibility of zero reflection of flexural waves from a tip of an ideal quadratic wedge. Note that a quadratic wedge provides the required minimum linear decrease in flexural wave velocity towards a sharp edge.

Let us consider the simplest one-dimensional case of plane flexural wave propagation in the normal direction towards the edge of a free elastic wedge (see Fig. 1) described by a power-law relationship $h(x)= \varepsilon x^m$, where $m$ is a positive rational number and $\varepsilon$ is a constant. Since flexural wave propagation in such wedges can be described in