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Understanding the dynamics of multi-degree-of-freedom nonlinear systems using backbone curves

D. J. Wagg[a,*]

[a]Department of Mechanical Engineering, University of Sheffield, Sheffield, S1 3JD, UK.

Abstract

In this paper we will describe how backbone curves can be used to explain complex dynamic phenomena that can occur in coupled multi-degree-of-freedom physical systems. Three examples will be used to demonstrate some key points. We will describe cases when backbone curves can be decoupled. In the case of nonlinear resonance (or modal interaction) we explain how to distinguish how many modes are interacting, their unison and relative phase characteristics. Bifurcation of higher order interaction curves from the lower order curves will also be discussed. Finally we will consider an example based on the transverse vibration of a thin plate with pinned boundary conditions. Both finite element simulations and a low order differential equation model are developed for this system. The results show the importance of the nonlinear coupling terms in replicating the frequency shift phenomena which is known to occur in structures of this type. Despite its much smaller size, the low order model is able to show qualitative agreement with the finite element model. Knowledge of the backbone curve behaviour for this system, is used to explain the forced damped behaviour.

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Keywords: backbone, vibration, nonlinear

1. Introduction

Traditionally modelling the dynamics of structures has been carried out using linear analysis, typically following the analytical approach first published by Lord Rayleigh [1]. When the behaviour of an engineering system is linear, computer simulations can be used to make accurate predictions of its dynamic behaviour, although we note that in some situations, particularly with complex geometries, even linear dynamic modelling can be difficult. The usefulness of linear theory is due, in large part, to the remarkable property of linear superposition, whereby a dynamic response for a structure can be obtained by adding together individual responses associated with sub-components of the response. Broadly speaking this approach has become known as "modal analysis" [2], where each mode of vibration is related to the physical configuration of the system and a corresponding resonant (or natural) frequency. Considerable effort has been made in recent years to extend this modal concept to the nonlinear domain, with the result that we now have a theory of nonlinear normal modes—see [3,4] and references therein.

* Corresponding author.
E-mail address: David.wagg@sheffield.ac.uk

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As we will discuss in this paper, one geometrical interpretation of a nonlinear normal mode is a so called backbone curve (also sometimes previously called a skeleton curve, although this terminology seems to have fallen out of use). This is a curve that relates the amplitude of the displacement to frequency. We will discuss how this concept can be used to model dynamic behaviour of nonlinear multi-degree-of-freedom (or coupled) oscillators. Furthermore, we will give examples of how backbone curves can be used to understand nonlinear resonance effects, where one part of a nonlinear system interacts with another part.

2. Backbone curves

The concept of a backbone curve is shown in Figure 1. Here, the backbone curve is shown as a long dash line starting at a Frequency value of 1. The main idea is that, for most structures, resonances lead to the largest vibration

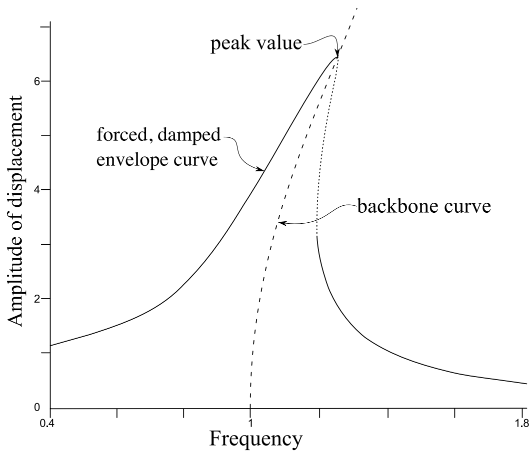

Fig. 1. The concept of a backbone curve, shown as the long dash line, for a single resonance peak. The solid line is stable forced, damped response, and the short dash line is the unstable response.

amplitudes, and so we want to understand the resonant response of the system. This is relatively straightforward for a linear system, because the resonances occur independently of each other, and respond only to external sources of excitation. However, for nonlinear systems, there is the possibility that resonances (often called "modes" in this context) can interact, due to nonlinear coupling between different parts of the structure. For example, membrane stresses (or axial loads) can provide this type of coupling in structures, as we will discuss in the final example of this paper.

So, when there are multiple degrees-of-freedom, backbone curves can be used to help understand the complexities of the resonant behaviour. This is because the resonant response of the forced system is closely linked to the unforced response, and a backbone curve represents the unforced, undamped response. The reasons to use them are that (i) they are generic in the sense that they have an influence on all forced-damped cases, and (ii) they are considerably easier to compute, than the forced, damped response for the system. One other point is worth noting, and that is that backbone curves are consistent with both linear and nonlinear multi-degree-of-freedom dynamics, unlike linear modes based on eigenvalues. So as systems get more complex, we can get physical insight into the resonant behaviour, by studying the backbone curves.

This idea is based on some key assumptions:

1. The damping is light, such that near resonance the forced, damped system is close to (i.e a small perturbation away from) the undamped system
2. The forced-damped envelope curve crosses the backbone curve at, or very close to, a bifurcation point i.e. in Figure 1 the crossing happens very close to the saddle-node bifurcation indicated at the peak value
3. The analysis is restricted to periodic orbits