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Application of Non-classical Shells Theories for Free Vibration Analysis of Annular Plates

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Abstract

In the present paper, the problem of evaluation natural frequencies of a transversely isotropic circular plate is considered and the impact of the material properties of the plate on its natural frequencies is studied. The classical Kirchhoff–Love (KL) theory only takes into account material properties of the midplane. That is why the fundamental frequencies for isotropic and transversely isotropic plates are equal for the classical theory. The Ambartsumyan (A) theory of anisotropic shells takes into account the impact of the shear deformation in the thickness direction on the stress-strain state of the plate. In the general case, the theory of anisotropic plates and shells developed by Rodionova, Titaev, and Chernykh (RTC) permits one to take into account not only the transverse shears, but also the deformation of normals to the midplane. In the present paper, the problem of determining the fundamental frequencies is solved both with A and RTC theories, which improve the KL theory. To study the impact of the radial inhomogeneity on the plate fundamental frequencies, the calculations are carried out using the commercial finite element software COMSOL Multiphysics (v. 5.0). It is shown that the inhomogeneity of the plate affects greatly the fundamental (lowest) frequency of the plate, while the plate non-isotropy has a greater influence on the second and higher vibration modes

Keywords:
Circular plate; transversely isotropic plate; vibrations

1. Introduction

The main purpose of the research is the mathematical modeling of the Lamina Cribrosa (LC), the part of a sclera, where optic nerve fibers pass through and where the layer of sclera becomes thinner and many little pores appear. One can consider LC as inhomogeneous plate, which has a circular or annular form, (inner radius in different eyes is different, but it changes from 0.05 R till 0.3 R, where R is outer radius). For some medical procedure it’s important to understand the sensitivity of the LC to the vibrations.

Transverse free vibrations of transversely isotropic inhomogeneous plates are considered in the research. It is known [1] that the Kirchhoff–Love (KL) classical theory of plates and shells is indifferent to changes of elastic properties in direction normal to the midsurface. Due to this fact the transverse shear strains should be taken into

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account when one wishes to refine the classical shell theory. The importance of the correction increases with the value of the ratio E_i/G_{i3} (E_i — Young's modulus in the i-th direction, G_{i3} — the transverse shear modulus) and the correction may be significant for highly anisotropic plates [1–3]. In its turn the inhomogeneity of a plate may be caused either by the change of the Young modulus in the radial direction or by the central circular hole or by both.

Free vibrations of circular plates made of inhomogeneous materials are analyzed with the help of different shell theories:

i) the Kirchhoff–Love (KL) theory,
ii) the refined theory by Ambartsumyan (A),
iii) the refined theory by Rodionova, Titaev, and Chernykh (RTC), and
iv) numerically with the 3D theory.

Theories ii) and iii) and for anisotropic plates and plates of moderate thickness [1, 2] take into account normal and rotary inertias.

We examine the effect of the material and geometrical properties of plates, including the influence of transverse shear modulus and material inhomogeneity and the size of the inner radius of the plate on the vibration modes and natural frequencies. The analytical results obtained with the help of the thin plates theories are compared with the 3D results obtained by FEM with the help of commercial finite element software COMSOL Multiphysics (v. 5.0).

2. Statement of the problem

The problem of determining the natural frequencies of a transversely isotropic circular plate with radius R and thickness h is considered. The material of the plate obeys Hookes law [2], assuming the plate midplane to be the plane of isotropy as follows:

$$\sigma_{ii} = E_{ii}e_{ii} + E_{ij}e_{jj} + E_{ik}e_{kk}, \;\; i \neq j \neq k, \qquad \sigma_{ij} = G_{ij}e_{ij}, \;\; i \neq j,$$

where

$$E_{11} = \frac{E_1(E_1 - E_3\nu_{13}^3)}{E^*}, \; E_{12} = \frac{E_1(E_1\nu_{12} + E_3\nu_{13}^2)}{E^*}, \; E_{13} = \frac{E_1E_3\nu_{13}(1-\nu_{12})}{E^*},$$
$$E_{33} = \frac{E_1E_3(1-\nu_{12}^2)}{E^*}, \; E^* = (1+\nu_{12})(E_1(1-\nu_{12}) - 2E_3\nu_{13}^2).$$

Here, σ_{ij} and e_{ij} are stresses and deformations respectively, $E_i, (i = r, \theta, z)$ is the Young modulus in the i-th directions; (r, θ, z) is the introduced cylindrical coordinate system, G_{ij} is the shear modulus in $(i - j)$ plane, ν_{ij} is Poissons ratio. For a transversely isotropic plate, $G_{12} = E_1/2(1 + \nu_{12})$, and $G_{13} = G_{23}$.

Firstly, we model the motion of a transversely isotropic plate by the Ambartsumyan refined theory [1]. The main assumptions of this theory are:

- The displacement w normal to the plate middle plane does not depend on the z-coordinate;
- The shear stresses σ_{rz} and $\sigma_{\theta z}$ change according to a given law with respect to the plate thickness

$$\sigma_{rz} = \frac{1}{2}\left(\frac{h^2}{4} - z^2\right)\varphi(r,\theta) \qquad \sigma_{\theta z} = \frac{1}{2}\left(\frac{h^2}{4} - z^2\right)\psi(r,\theta),$$

where φ and ψ are the unknown functions of coordinates r, θ.

Due to these assumptions we have for displacements

$$u_z = w(r), \quad u_r = u - z\frac{\partial w}{\partial r} + \frac{z}{2}\left(\frac{h^2}{4} - \frac{z^2}{3}\right)\frac{\varphi(r,\theta)}{G_{13}}, \quad u_\theta = v - z\frac{\partial w}{r\partial\theta} + \frac{z}{2}\left(\frac{h^2}{4} - \frac{z^2}{3}\right)\frac{\psi(r,\theta)}{G_{13}},$$

where u and v are the displacement components in the tangential plane.