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Application of the dynamic stiffness method in the vibration analysis of stiffened composite plates

Emilija Damnjanović[a], Marija Nefovska-Danilović[a]*, Mira Petronijević[a], Miroslav Marjanović[a]

[a]Faculty of Civil Engineering, University of Belgrade, Bulevar kralja Aleksandra 73, 11000 Belgrade, Serbia

Abstract

Composite laminates are nowadays extensively applied in many engineering disciplines. Free vibration characteristics of such structures are not always easy to predict by using conventional finite element method (FEM). As an alternative, the dynamic stiffness method (DSM) can be applied to predict free vibration characteristics of composite plate assemblies, especially in mid and high frequency ranges. Key feature of the DSM is the dynamic stiffness element (DSE) and its dynamic stiffness matrix, derived from the exact solution of the governing equations of motion in the frequency domain. Consequently, the structural discretization is influenced only by the change in the geometrical and/or material properties of the structure. The number of unknowns is significantly decreased in comparison with the FEM, without losing the accuracy and reliability of the results.
In the paper, the DSE based on the higher order shear deformation theory (HSDT) is applied to study free vibration analysis of composite stiffened plates. The numerical analysis has been carried out through an illustrative example in order to check the accuracy of the proposed method. The influence of side-to-thickness ratio on the free vibration characteristics of stiffened plate has been studied numerically. The results are validated using the available analytical data as well as with the FEM solutions.

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Keywords: dynamic stiffness method; stiffened plate; composite laminate; HSDT.

1. Introduction

Composite laminates are nowadays extensively applied in many engineering disciplines as structural components of aircraft wings, ship hulls and FRP bridges, amongst others. In these structures, usually constructed in the form of

* Corresponding author. Tel.: +381-11-3218-552; fax: +381-11-3370-223.
E-mail address: marija@grf.bg.ac.rs

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plates joined at different angles, both in-plane and bending modes of vibration have been coupled. Free vibration characteristics of such structures are not always easy to predict by using conventional finite element method (FEM) based on the polynomial shape functions and weak form solution of the corresponding elastodynamic problem. As an alternative, the dynamic stiffness method (DSM) can be applied to accurately and efficiently predict the free vibration characteristics of composite plate assemblies, especially in the mid and high frequency ranges. The DSM is referred as the strong form-based method formulated in the frequency domain. Key feature of the DSM is the dynamic stiffness element and the corresponding dynamic stiffness matrix derived from the exact solution of the governing equations of motion. Consequently, the structural discretization is influenced only by the change in the geometrical and/or material properties of the structure. The number of unknowns is decreased in comparison with the FEM, decreasing the computational time/memory cost without losing the accuracy and reliability of the results.

Initially, application of the DSM was limited to the free vibration analysis of one-dimensional structures (beams and bars) [1-4] and Levy-type plates having two opposite edges simply supported [5-9], for which the governing equations of motion could be solved analytically. For plates having arbitrary boundary conditions, the issues of assignment of continuous boundary conditions and analytical solution of the governing equations, have occurred. The above issues can be overcome by using the projection method, presented in the works of Kevorkian & Pascal [10] and Casimir et al. [11]. Recently, in a series of contributions, a number of dynamic stiffness elements have been developed and applied in the free vibration analysis of plate assemblies undergoing both in-plane [12-15] and transverse vibration [16-22], accounting for different plate theories, material orthotropy, multi-layer properties and arbitrarily assigned boundary conditions.

In this paper, previous authors' work [12, 19, 21-22] has been extended and applied in the free vibration analysis of composite stiffened plates with arbitrary boundary conditions. First, the dynamic stiffness matrix of composite plate element undergoing in-plane vibration has been developed. Afterwards, the developed dynamic stiffness matrices for both in-plane and transverse vibration [21] based on the higher order shear deformation theory (HSDT) are rotated using the rotation matrix, before they have been assembled into the global dynamic stiffness matrix of the stiffened plate. The numerical analysis has been carried out through an illustrative example in order to check the accuracy of the proposed method. The influence of side-to-thickness ratio has been studied. The results have been validated against the available analytical data as well as against the FEM solutions.

2. Dynamic stiffness formulation of the dynamic stiffness element undergoing in-plane vibration

In the paper, we consider an assembly of rectangular cross-ply (0/90) laminated composite plates, each having the dimensions $2a\times2b$ and being composed of n orthotropic layers. The HSDT implies the following assumptions: (i) all layers are perfectly bonded together, (ii) the material of each layer is homogeneous, orthotropic and linearly elastic, (iii) small strains and small rotations are assumed, (iv) inextensibility of the transverse normal is imposed, (v) the displacement field is approximated using a cubic variation of the in-plane displacements through the thickness of the plate, leading to more realistic warping of the cross section and quadratic variation of transverse shear strains and transverse shear stresses through each layer of the laminate, [23]. The previous assumptions eliminate the application of the shear correction factors.

The formulation of the dynamic stiffness element is conducted separately for the in-plane and transverse vibration of the laminated composite plate, starting from two independent sets of Euler-Lagrange equations of motion. While the dynamic stiffness matrix of composite plate undergoing transverse vibration has been already formulated in authors' previous work [21], the procedure for the development of the dynamic stiffness matrix of rectangular laminated composite plate element undergoing in-plane vibration will be briefly presented, as follows.

In the first step, the harmonic representation of the in-plane displacement components u_0 and v_0 is introduced:

$$u_0\left(x,y,t\right)=\sum\hat{u}_0\left(x,y,\omega\right)e^{i\omega t},\quad v_0\left(x,y,t\right)=\sum\hat{v}_0\left(x,y,\omega\right)e^{i\omega t} \tag{1}$$

where $\hat{u}_0$ and $\hat{v}_0$ are the amplitudes of the in-plane displacement components u_0 and v_0, defined in the frequency domain, while ω is the considered angular frequency. Having in mind that Eq. (1) is valid for all angular frequencies in the considered frequency range, the argument ω will be omitted in further representations.