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Asymptotic and numerical analysis of free low-frequency ring-stiffened shells vibrations

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Abstract

Small free low-frequency vibrations of ring-stiffened cylindrical shells are considered. This problem is reduced to solution of the eigenvalue problems of linear differential equations. The equations describing the vibration of thin shells contain the dimensionless shell thickness as a small parameter. It allows to find the solution of the initial eigenvalue problem as the sum of slowly varying functions and edge effect integrals. Thus the initial system of differential equations is transformed into an approximate system of the smaller order. The simple asymptotic formulas for low frequencies are derived. Numerical results are obtained with the help of a shooting procedure. The asymptotic and numerical results converge.

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Keywords: Free vibrations; Ring-stiffened shell; Eigenvalue problems; Asymptotic methods; Shooting procedure

1. Introduction

Thin shells are widely applied in engineering. The important characteristic of such shells is its fundamental vibration frequency. A simple way to raise the fundamental frequency to avoid resonance is to increase the thickness of the shell. However, in this case, the mass of the shell also increases. That is why to make frequencies higher, the stiffening rings are frequently used. In the paper [1] it is shown that replacement of an unstiffened cylindrical shell by an optimal ring-stiffened shell with the same mass raises the fundamental vibration frequency in some times.

Various methods for the analysis of ring-stiffened shells vibrations have been developed. In [2] natural frequencies of stiffened shells are obtained by the Fourier expansions. Rayleigh-Ritz procedure was used in [3] for vibration analysis of ring-stiffened cylindrical shell. The study [4] has shown that the finite element method is quite suitable to analyze the vibration characteristics of ring-stiffened cylindrical shells. The application of asymptotic methods to the problems of free low-frequency ring-stiffened shells vibrations are presented in [1, 5]. The benefit of using an asymptotic approach is that asymptotic results clarify the dynamic behavior of ring-stiffened shells.

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Almost in all studies of ring-stiffened shells, including [1–4], rings have been considered as circular beams. However, the wide rings must be treated as annular plates. In [5] vibrations of a cylindrical shell stiffened at one edge by an annular plate were studied. In this paper we assume that n_r identical annular plates are located on internal parallels of the shell (see Figure 1a, where $n_r = 5$).

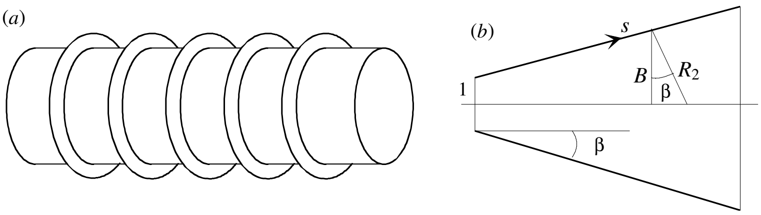

Fig. 1. (a) ring-stiffened cylindrical shell; (b) conical shell.

2. Basic equations

After separation of variables the non-dimensional equations describing free vibrations of a conical shell can be written in the following form

$$
\begin{aligned}
&T_1' + \frac{B'}{B}(T_1 - T_2) + \frac{m}{B}S + \lambda u = 0, \quad S' + 2\frac{B'}{B}S - \frac{m}{B}T_2 + \frac{Q_2}{R_2} + 2H' + \lambda v = 0,\\
&Q_1' + \frac{B'}{B}Q_1 + \frac{m}{B}Q_2 - \frac{T_2}{R_2} + \lambda w = 0, \quad Q_1 = M_1' + \frac{B'}{B}(M_1 - M_2) + 2\frac{m}{B}H, \quad Q_2 = -\frac{m}{B}M_2 + 2\frac{B'}{B}H,\\
&M_1 = \mu^4(\kappa_1 + \nu\kappa_2), \quad M_2 = \mu^4(\kappa_2 + \nu\kappa_1), \quad H = \mu^4(1-\nu)\vartheta_2',\\
&T_1 = \varepsilon_1 + \nu\varepsilon_2, \quad T_2 = \varepsilon_2 + \nu\varepsilon_1, \quad S = \frac{1-\nu}{2}\left(v' - \frac{B'}{B}v - \frac{m}{B}u\right),\\
&\varepsilon_1 = u', \quad \varepsilon_2 = \frac{m}{B}v + \frac{B'}{B}u + \frac{w}{R_2}, \quad \kappa_1 = \vartheta_1', \quad \kappa_2 = \frac{m}{B}\vartheta_2 + \frac{B'}{B}\vartheta_1, \quad \vartheta_1 = -w', \quad \vartheta_2 = \frac{m}{B}w + \frac{v}{R_2},
\end{aligned}
\tag{1}
$$

where (′) denotes the derivative with respect to the longitudinal coordinate s; m is circumferential wave number; $\lambda = 4\pi^2\sigma\rho f^2 R^2 E^{-1}$ is the frequency parameter; $\sigma = 1 - \nu^2$, ν is Poisson's ratio, E is Young's modulus; ρ is the mass density; f is the vibration frequency; μ is small parameter, $\mu^4 = h^2/12$; h is the dimensionless shell thickness.

Variables T_1, T_2 and S are the stress-resultants in the neutral surface; Q_1 and Q_2 are the transverse shear forces; M_1 and M_2 are the bending moments, H is the twisting moment; ϑ_1 and ϑ_2 are the angles of rotation of the normal; u, v, w are the projections of displacement. The directions of forces and moments are presented in [6].

The minimal radius of the truncated cone is taken as a characteristic length. The function $B(s) = 1 + s\sin\beta$ is the dimensionless distance between the point on the middle surface and the axis of symmetry, $R_2(s) = B(s)/\cos\beta$ is the main radius of curvature, β is the angle between the axis of the cone and its generator (see Figure 1b).

In the case $\beta = 0$, $B = R_2 = 1$ equations (1) describe free vibrations of a cylindrical shell. If $\beta = \pi/2$ then $B = 1+s$, $1/R_2 = 0$ and system of equations (1) splits into two systems, corresponding transverse flexural and tangential (in plane) vibrations of an annular plate. For the cylindrical shell $s \in [0, l]$, where l is the dimensionless length of the shell. For the annular plate $s \in [1, 1 + b]$, where b is the dimensionless width of the plate. In equations (1) for the plate it is necessary to replace the thickness of the shell h by the thickness of the plate a.

Let s_i, $i = 1, 2, \ldots n_r$ are the coordinates of the shell parallels stiffened by rings and $s_0 = 0$, $s_n = l$, $n = n_r + 1$. We denote as $u^{(j)}$, $v^{(j)}$, $w^{(j)}$,… the solutions of equations (1) for the shell in the intervals $s \in [s_{j-1}, s_j]$, $j = 1, 2, \ldots n$. The solutions of equations (1) for the plates located on parallel with the coordinate $s = s_i$ are $u_p^{(i)}$, $v_p^{(i)}$, $w_p^{(i)}$,….

We assume that the shell and the plates are made of a same material. Then at the parallel $s = s_i$ the following 12 continuity conditions have to be satisfied

$$
\begin{aligned}
&w^{(i)} = w^{(i+1)} = -u_p^{(i)}, \quad u^{(i)} = u^{(i+1)} = w_p^{(i)}, \quad v^{(i)} = v^{(i+1)} = v_p^{(i)}, \quad \vartheta^{(i)} = \vartheta^{(i+1)} = \vartheta_p^{(i)},\\
&h(T_1^{(i+1)} - T_1^{(i)}) = aQ_{1p}^{(i)}, \quad h(S^{(i+1)} - S^{(i)}) = aS_p^{(i)}, \quad h(M_1^{(i+1)} - M_1^{(i)}) = aM_{1p}^{(i)}, \quad h(Q_1^{(i+1)} - Q_1^{(i)}) = -aT_{1p}^{(i)},
\end{aligned}
\tag{2}
$$