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Computational method for the dynamics of railway tracks on a non-uniform viscoelastic foundation

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Abstract

The foundation of a railway track may be non-uniform due to a number of reasons and this area is currently the subject of significant research. In this article, a new method has been developed for computing the responses of railway tracks based on a non-uniform foundation subjected to moving forces. This method is a coupling of an analytical model for the rail together with the sleepers and a finite element method for the foundation. In steady-state, it is supposed that the responses are unchanged when the moving forces come and go away from a larger interval of the railway track which contains a non-uniform zone. The dynamical stiffness matrix (DSM) of the foundation is computed by the finite element method and it is transformed to meet the steady state boundary condition. On the other hand, the rail together with the sleepers and rail pads are modelled by a periodically supported beam subjected to moving forces. This analytical model leads to a relation between the reactions forces and the displacements of the sleepers. This relation describes also the degrees of freedom (DOFs) of the nodes of the foundation at the contact surfaces with the sleepers. Then, a transformation technique has been developed in order to substitute the analytical relation into the DSM. Finally, the responses have been computed by using the transformed DSM. This method is a coupling of the analytical and numerical methods. Therefore, it has reduced all DOFs of the track components (sleepers, rail, and rail pads) which gives a significant advantage in computational time.

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Keywords: Railway track; transition zone; non-uniform foundation; periodically supported beam; structural dynamics.

1. Introduction

The influence of a non-uniform foundation on the response of a railway track has been studied by different methods including [1–6]. The most pressing difficulty of the numerical methods is that the rail with its supports (sleepers) and the foundation are not of the same scale (the dimensions of foundation is much larger than ones of sleepers and rails) which increases the degrees of freedom (DOF) and costs computing time. Recently, some authors have developed different techniques to reduce the number of DOF.

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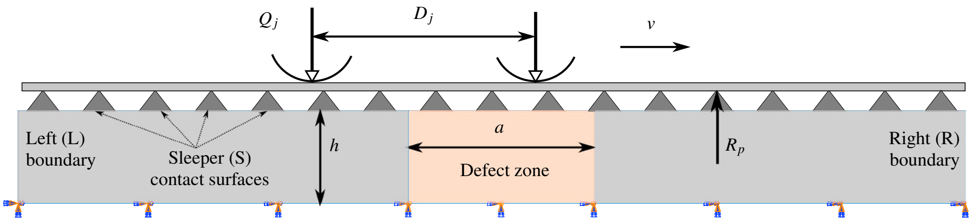

Fig. 1. Coupling of analytical and numerical models

This article presents a new model which is a coupling of analytical and numerical methods. When the rail together with its supports is considered as a periodically supported beam, Hoang et al. [1,7] proved a relation between the support displacements and reaction forces in the steady state and this relation holds for all types of foundation. By using this analytical model, we can write this relation for all DOF at the contact surfaces between the sleepers and the foundation. Then, this relation is substituted in the finite element model of the foundation in order to get the dynamical response.

2. Formulations

Let us consider a railway track based on a visco-elastic foundation which contains a defect zone as shown in Figure 1. In this model, we consider the rail as an infinite beam subjected to moving forces and the sleepers are concentrated supports which are distributed periodically along the beam (the rail together with its supports is called a periodically supported beam subjected to moving forces). Otherwise, the foundation is a 2D visco-elastic mater which is modelled by the finite element method.

Nomenclature

$\mathbf{u}$	vector of nodal displacements
$\mathbf{f}$	vector of nodal forces
$\mathbf{D}$	dynamical stiffness matrix of the foundation
$_S$	denotes the foundation DOF at the contact surfaces with the sleepers
$_L$	denotes the foundation DOF at the left boundary
$_R$	denotes the foundation DOF at the right boundary
$_I$	denotes the other foundation DOF
ω	angular frequency
$\mathbf{Q}_e$	equivalent force of the periodically supported beam
$\mathbf{K}_e$	equivalent stiffness of the periodically supported beam

By using the finite element method we can obtain the following results from the dynamic equation of the foundation

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{F}(t) \tag{1}$$

where $\mathbf{M}, \mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices of the foundation, and $\mathbf{u}(t), \mathbf{F}(t)$ are the nodal displacements and forces. We can write the aforementioned equation in the frequency domain

$$\left(-\omega^2\mathbf{M} + \mathrm{i}\omega\mathbf{C} + \mathbf{K}\right)\mathbf{u}(\omega) = \mathbf{F}(\omega) \tag{2}$$

or

$$\mathbf{D}(\omega)\mathbf{u}(\omega) = \mathbf{F}(\omega) \tag{3}$$