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Application of differential transformation finite element method in aperiodic vibration of non-prismatic beam

Ryszard Hołubowski*

Faculty of Civil Engineering, Wrocław University of Science and Technology, 27 Wyb. Wyspiańskiego St, 50-370 Wrocław, Poland

Abstract

The paper presents the combination of multi-step differential transformation method (MsDTM) and finite element method (FEM) in vibration analysis of non-prismatic beam. A new algorithm, named differential transformation finite element method (DTFEM), combines the advantages of both methods. It enables to solve a wide class problems and obtain solutions in the form of piecewise functions that are very useful in further analyses, especially in time-consuming computations of dynamic systems. In contrary to standard DTM, with the proposed method, we can successfully calculate vibrations of non-prismatic structures governed by partial differential equations with variable coefficients. The application of FEM reduces partial differential equation to system of ordinary differential equations depending on time. After applying multi-step DTM the system is converting into a set of recursive algebraic equations which can be solved in a symbolic way. The final approximate solution is composed of a sequence of single solutions obtained in every time interval.

The algorithm of DTFEM was described and illustrated with an example of non-prismatic simply supported beam subjected to uniformly distributed load increasing nonlinearly in time. The results in the form of beam displacements, velocities and accelerations were compared with numerical approach based on Newmark method. Two cases of damping model were taken into account: structural damping model and external damping model represented by two viscous dampers. A very good agreement was found in both cases. Presented analyses show that DTFEM is more accurate, with the same time step, than Newmark method. Therefore, in some cases, it may be possible to obtain sufficiently accurate results faster. The main advantage of the proposed method is the solution's form which is a function of time and not a discrete set of results.

Keywords: Differential transformation finite element method; aperiodic vibration; damped vibration; non-prismatic beam.

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* Corresponding author. Tel.: +48-71-320-48-58.
E-mail address: ryszard.holubowski@pwr.edu.pl

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1. Introduction

In numerous publications on vibration of structures the most attention is paid to natural or free vibration analysis. Authors relatively rarely describe new methods that allow to obtain the response of a structure subjected to time varying load, especially when the structure is non-prismatic and the load is aperiodic. The reason for this state of affairs is difficulty with solving governing equations which in this case are partial differential equations with variable coefficients. In most commonly used methods, like Newmark, Wilson or Houbolt, time histories of forced vibrations are the result of numerical integration. The essential drawback of such approach is the solution's form which is a discrete set of results and not a function of time.

A novel algorithm of differential transformation finite element method (DTFEM), presented in this paper, allows to overcome this problem. The DTFEM was described for the first time by Hołubowski and Jarczewska in [1] on the example of undamped forced vibrations of non-prismatic beam. The method is based on combination of well-known finite element method (FEM) and multi-step differential transformation method (MsDTM) proposed by Odibat *et al.* [2], which, in turn, is a modified version of classical differential transformation method (DTM) introduced by Pukhov and Zhou [3, 4, 5, 6]. The MsDTM was successfully used to solve a problem of a third grade non-Newtonian fluid flow between two parallel plates [7], a problem of temperature fluctuations in the western and eastern parts of the equatorial ocean governed by Vallis models [8] and many other nonlinear problems described by Lotka-Volterra, Chen and Lorenz systems [2], or Riccati, Duffing and van der Pol equations [9].

2. Differential transformation finite element method

The idea of the method, described in details in [1], will be presented on the example of transverse vibrations of non-prismatic beam loaded by arbitrarily time-varying load represented by function $P(x, t)$. The dynamic response $w(x, t)$ satisfies the equation

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 w(x,t)}{\partial x^2}\right]+m(x)\frac{\partial^2 w(x,t)}{\partial t^2}=P(x,t) \tag{1}$$

where $EI(x)$ and $m(x)$ are the bending stiffness and mass per unit length.

In the first step, the partial differential equation (1) is converted into a set of ordinary differential equations by using FEM. After finite element discretization, the approximate solution on the element's length l_e is given by

$$w(\xi,t)=\mathbf{W}_e^{\mathrm{T}}(t)\mathbf{N}_e(\xi)\,, \tag{2}$$

where $\mathbf{W}_e = [w_i, \varphi_i, w_j, \varphi_j]^{\mathrm{T}}$ is nodal displacement vector of the element *i-j*, $\mathbf{N}_e$ is vector of shape functions, subscript e denotes the number of beam element, $[\ ]^{\mathrm{T}}$ is the transposition operator and $\xi = x/l_e$, $0 \le \xi \le 1$. After formulating energy expressions, applying Lagrange equations and using aggregation formula we obtain the following matrix equation

$$\mathbf{B}\ddot{\mathbf{q}}(t)+\mathbf{K}\mathbf{q}(t)=\mathbf{F}(t)\,, \tag{3}$$

which can be supplemented by a damping component proportional to the velocity of beam displacements listed in vector $\mathbf{q}(t)$

$$\mathbf{B}\ddot{\mathbf{q}}(t)+\mathbf{C}\dot{\mathbf{q}}(t)+\mathbf{K}\mathbf{q}(t)=\mathbf{F}(t)\,. \tag{4}$$

The vector $\mathbf{F}(t)$ represents excitation forces, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{K}$ are mass, damping and stiffness matrices, respectively.