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Procedia Engineering 199 (2017) 378–383

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

Model order reduction in design of parameterized structures under different load configurations

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Abstract

In structural design of mechanical systems a dynamic analysis is carried out in the time domain or in the frequency domain which implies solving the equation of motion several times. Usually the systems depend on a set of parameters which influence their responses. Thus the design process includes numerical simulations using a full-scale finite element (FE) model for each set of parameters which is computationally demanding and time consuming. In this contribution the response in the frequency domain due to different load configurations is investigated by using a mixed approach of two related methods for parametric model order reduction (MOR) based on interpolation in matrix manifolds of the reduced order models (ROMs) and by using a global basis over the parametric space. Furthermore an approach based on interpolation of the reduced solution is presented. These approaches of MOR permit the computational efficient evaluation of different load configurations and avoid the generation of a new FE model for each case. A numerical example illustrates the capability of those methods. The respective results using parametric MOR (pMOR) approaches are compared with the solution obtained by using the corresponding full-scale FE model and the direct application of the Krylov subspace method (KSM).

Keywords:
Parametric model order reduction, Interpolation in matrix manifolds, Frequency response, Krylov subspace, Interpolation

1. Introduction

The equation of motion resulting from a finite element discretization of a mechanical system is given by

$$\mathbf{M}\ddot{u}(t) + \mathbf{D}\dot{u}(t) + \mathbf{K}u(t) = f(t) \tag{1}$$

Applying a Fourier transformation to Eq.1, with $\mathbf{u}(\omega) = \mathcal{F}\big(u(t)\big)$ and $\mathbf{f}(\omega) = \mathcal{F}\big(f(t)\big)$ leads to

$$(-\omega_j{}^2\mathbf{M} + i\omega_j\mathbf{D} + \mathbf{K})\mathbf{u}(\omega_j) = \mathbf{f}(\omega_j) \quad j = 1, 2, 3...n_\omega \tag{2}$$

which is the equation of motion in the frequency domain, where $\mathbf{M}, \mathbf{D}, \mathbf{K} \in \mathbb{R}^{N\times N}$ are the mass, damping and stiffness matrix respectively, $\mathbf{f} \in \mathbb{R}^{N\times 1}$ is the load vector (force vector) and $\mathbf{u} \in \mathbb{R}^{N\times 1}$ is the displacement vector. A reduced-order model (ROM) which would lead to a lower computational time is achieved by using a suitable MOR technique.

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1877-7058 © 2017 The Authors. Published by Elsevier Ltd.
Peer-review under responsibility of the organizing committee of EURODYN 2017.
10.1016/j.proeng.2017.09.049

The main idea of MOR techniques is to find a vector space spanned by the columns of $\mathbf{V} \in \mathbb{C}^{N \times n_r}$, with $n_r \ll N$, which maps a reduced set of degrees of freedom (dofs) $\mathbf{u}_r \in \mathbb{C}^{n_r \times 1}$ into the original set of degrees of freedom $\mathbf{u}$, such that

$$\mathbf{u} \approx \tilde{\mathbf{u}} = \mathbf{V}\mathbf{u}_r \tag{3}$$

The reduced order model is given by

$$(-\omega_j{}^2\mathbf{M}_r + i\omega_j\mathbf{D}_r + \mathbf{K}_r)\mathbf{u}_r(\omega_j) = \mathbf{f}_r(\omega_j) \tag{4}$$

where $\mathbf{M}_r, \mathbf{D}_r, \mathbf{K}_r \in \mathbb{C}^{n_r \times n_r}$, $\mathbf{f}_r \in \mathbb{C}^{n_r \times 1}$ are defined by

$$\begin{aligned} &\mathbf{M}_r = \mathbf{V}^H\mathbf{M}\mathbf{V} \quad \mathbf{D}_r = \mathbf{V}^H\mathbf{D}\mathbf{V} \quad \mathbf{K}_r = \mathbf{V}^H\mathbf{K}\mathbf{V} \\ &\mathbf{f}_r = \mathbf{V}^H\mathbf{f} \end{aligned} \tag{5}$$

For the construction of that matrix $\mathbf{V}$ different approaches have been proposed in the field of structural mechanics, i.e. Real Modal Analysis, Guyan-Irons Reduction, Improved Reduction System, Dynamic Reduction, Craig-Bampton Method, Krylov subspace method and Derivative-based Galerkin Projection. A review of those MOR techniques is presented in [1].

When Eq. 2 depends on a set of parameters $\pi_i \in \mathbb{R}^k$ it is possible to have n_π ROMs corresponding to n_π sets of parameters, but the question arises how to use those ROMs in order to determinate the solution in a new parameter set $\pi_{n_\pi+1}$. Interpolation on Matrix Manifolds is a promising approach because it works only at the level of ROMs. In this contribution those methods are applied to a system with different load configurations. The results are compared with results from a ROM using a truncated global basis $\mathbf{V}_g$ and the one from interpolation of the reduced solutions. In the next section the theoretical background for each approach is summarized. In section 3 a numerical example illustrates the capabilities of the pMOR mentioned above and a conclusion is given in section 4.

2. Parametric Model Order Reduction

2.1. pMOR based on interpolation in matrix manifolds

The two main approaches for the pMOR based on interpolation in Matrix manifolds are reviewed in this subsection, i.e. the method developed by Lohmann [2] (Method A) and the method developed by Farhat [4] (Method B). The general procedure to use method A and B was introduced by Geuss et. al [3]. Here the general procedure is summarized for the particular problem which is addressed in this work.

- Sample the parametric space. A coarse grid is defined in the parametric space of interest, i.e. n_π parametric configurations are chosen.
- Reduction of the local system. n_π ROMs are computed using any MOR technique and a data base is build with $\mathbf{M}_{ri}$, $\mathbf{D}_{ri}$, $\mathbf{K}_{ri}$, $\mathbf{F}_{ri}$, $\mathbf{V}_i$, for $i = 1...n_\pi$. In this contribution the KSM is used to reduced the systems at the samples points.
- Adjustment of the Matrix $\mathbf{V}$. The ROMs should be expressed in a common subspace $\mathbf{V}_x$, i.e. all the local systems share the same generalized coordinates $\mathbf{x}$. This is achieved using a transformation matrix $\mathbf{Q}_i$:

  $$\mathbf{V}_x\mathbf{x} = \mathbf{V}_i\mathbf{Q}_i\mathbf{x} \tag{6}$$

  The definitions of matrix $\mathbf{V}_x$ and $\mathbf{Q}_i$ according to Method A and B are given in table 1.
- Choice of the interpolation manifold. Method A uses the original matrix manifold for the interpolation while method B uses a tangent manifold as interpolation space. The projection to a tangent space to the matrix manifold is given by the logarithm mapping and to transform back to the original manifold the exponential mapping is used. The logarithm mapping of a matrix $\mathbf{M}_i$ with respect to $\mathbf{M}_x$ is given by [5]

  $$\mathbf{M}_{log,i}(\mathbf{M}_x, \mathbf{M}_i) = \mathbf{U}\mathbf{\Theta}\mathbf{V}^T \tag{7}$$