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ScienceDirect

Procedia Engineering 199 (2017) 471–476

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

Mass sensing using self-excited oscillation in viscous environments

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Abstract

A mass measurement method in viscous environments using self-excited oscillation is proposed theoretically and experimentally. The utilization of the natural frequency shift in a resonator has attracted many researchers due to the high sensitivity. Small mass measurement with the order of fg and ag has been reported. In the conventional methods, the natural frequency shift is detected from the shift of the excitation frequency at which the frequency response curve takes the peak value. However, it is not applicable to the measurement in high viscous environments, i.e., the observation of the chemical reaction and the mass sensing for biological samples because the peak is ambiguous or does not exist. In this research, as a mass measurement method applicable in viscous environments, we propose the utilization of self-excited oscillation to measure the shift of the natural frequency caused by an sample added to the resonator. The resonator is actuated by positive velocity linear feedback. Self-excited oscillation occurs when the linear feedback gain exceeds a critical value depending on the viscosity in measurement environments. Also, by applying the nonlinear feedback the divergence of the self-excited oscillation is suppressed and the response amplitude is kept constant. The self-excited resonator vibrates at the natural frequency regardless of the viscosity in measurement environments because the velocity feedback compensates for the viscous effect. Then, not relying on a frequency response curve, the method using natural frequency shift becomes applicable even in high viscous environments. We fabricated a theoretically proposed self-excited mass sensor and conducted mass measurements experimentally in air, in water, and in a liquid with 1.17×10^4 mPa · s viscosity. As a result, the mass of 0.0091 kg was detected in the respective viscous environments. The ratio of the additional mass to the mass of the sensor is 0.59 %.

Peer-review under responsibility of the organizing committee of EURODYN 2017.

Keywords: self-excited oscillation, mass measurement, viscous environment, velocity feedback

1. Introduction

Mass measurement methods using resonance frequency shift have attracted much attention in recent years due to its high sensitivity[1–5]. It is a method to estimate the additional mass from the change of the resonance frequency in a resonator. The resonance frequency is mainly derived from the frequency at which the frequency response curve peaks. However, there is difficulty in the application of this method in a viscous environment. It becomes difficult to measure the resonance frequency because the peak becomes ambiguous or disappears due to the viscous

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1877-7058
Peer-review under responsibility of the organizing committee of EURODYN 2017.
10.1016/j.proeng.2017.09.051

environment. Furthermore, because the sensitivity is proportional to the resonance frequency of the resonator, the sensitivity decreases by the decrease of resonance frequency due to the viscosity. These are problems for applications in liquid, for example, the observation of the chemical reaction and the mass sensing for biological samples.

To solve these problems, we propose to use the self-excited oscillation in the resonator for mass measurement instead of the resonance frequency under the forced excitation. When self-excited oscillation occurs, the force due to velocity feedback compensates for the viscous friction and it can be regarded as an undamped system. Therefore, by using the frequency shift of self-excited oscillation for mass measurement, it is expected that the mass measurement is not affected by the viscosity of the environment.

In this study, we clarify the condition that the self-excited oscillation occurs and derive the equation to measure the mass. We perform mass measurements experiment to confirm the validity of the proposed method in liquid environments.

2. Theoretical analysis of mass measurement method utilizing self-excited oscillation

In this section, we show the condition that the self-excited oscillation occurs and the equation of mass measurement method. The equation of motion of the resonator with one degree of freedom subjected to the viscous friction is given as

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0, \tag{1}$$

where x, m, c and k are displacement of the resonator, the mass of the resonator, the damping coefficient and the spring coefficient, respectively. To generate self-excited oscillation, the resonator is subjected to a force proportional to its own velocity[6,7] as shown in

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = c'\frac{dx}{dt}, \tag{2}$$

where c' is a variable gain. By combining the same terms, we obtain the equation

$$m\frac{d^2x}{dt^2} + (c - c')\frac{dx}{dt} + kx = 0. \tag{3}$$

The qualitative features of the motion represented by Eq. (3) change with the relationship of the magnitudes of c and c'. In particular, when $c - c' < 0$, the resonator becomes a negative damping system and the self-excited oscillation occurs. The response frequency of self-excited oscillation f is expressed as

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \frac{(c - c')^2}{4m^2}}. \tag{4}$$

Here, by adjusting c' to satisfy the following condition,

$$\frac{k}{m} \gg \frac{(c - c')^2}{4m^2} \Leftrightarrow c < c' \ll c + 2\sqrt{mk}, \tag{5}$$

the effect of viscosity becomes sufficiently small and the natural frequency f is expressed as

$$f \simeq \frac{1}{2\pi}\sqrt{\frac{k}{m}}. \tag{6}$$

As can be seen from Eq. (6), the response frequency of self-excited oscillation agree with the natural frequency of the resonator regardless of the viscosity when the condition of Eq. (5) is satisfied.

Next, we explain mass measurement method utilizing the self-excited oscillation. In the following discussion, the response frequency of the self-excited oscillation is expressed by Eq. (6). We add small mass Δm to the resonator. Letting that the response frequencies before and after adding Δm are f_1 and f_2 respectively. The difference between the two frequencies is expressed as

$$f_2 - f_1 = \frac{1}{2\pi}\left(\sqrt{\frac{k}{m + \Delta m}} - \sqrt{\frac{k}{m}}\right) \simeq -\frac{f_1}{2m}\Delta m. \tag{7}$$