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Geometrically nonlinear transverse vibrations of Bernoulli-Euler beams carrying a finite number of masses and taking into account their rotatory inertia

Adri Ahmed[a] , Benamar Rhali[b]

[a]*Laboratoire de Mécanique Productique & Génie Industriel (LMPGI) Ecole Supérieure de Technologie; Hassan IIUniversity in Casablanca, KM 7 Route El Jadida, Casablanca 20000, Morocco*
[b]*Equipe des Etudes et Recherches en Simulation et Instrumentation Ecole Mohammadia des Ingénieurs ; Mohammed V University in Rabat, Avenue Ibnsina,Agdal, Rabat, Morocco*

Abstract

The objective of this paper is to establish the theoretical formulation of the problem of nonlinear transverse vibrations of Bernoulli–Euler beams carrying a finite number of masses at arbitrary positions, with general end conditions. The generality of the approach is based on use of translational and rotational springs at both ends, allowing examination of all possible combinations of classical beam end conditions, as well as elastic restraints. The method used is based Hamilton's principle and spectral analysis for nonlinear free vibrations exhibiting large displacement amplitudes. The problem is reduced to solution of a nonlinear algebraic system using numerical or analytical methods. This has been previously applied to nonlinear transverse vibrations of continuous structures such as beams, plates and shells, to nonlinear longitudinal vibrations of 2-dof and multi-dof systems and to nonlinear transverse vibrations of N-dof systems. The nonlinear algebraic system has been solved using an approximate explicit method developed previously (The so-called second formulation) leading to the nonlinear fundamental mode shape of beams carrying a finite number of masses and to the corresponding backbone curves.

*Keywords:*Nonlinear vibration ; Rotary inertia ; Hamilton principle ; Backbone curve ; single mode approach

1. Introduction

Many real systems can be modeled by a beam supporting a finite number of masses. One can mention for example the wing of an airplane supporting the engine mass or a bridge supporting many masses or the axis of a machine supporting mechanical components like toothed wheels. On the other hand, a geometrically non-linear behavior occurs when a structure is subjected to high vibration amplitudes, especially in the neighborhood of one of its resonance frequencies, making it inaccurate to use classical linear theories to estimate the corresponding induced

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strains and stresses. In previously published works, the nonlinear mode shapes and resonance frequencies of beams with various end conditions have been examined both theoretically and experimentally [3 to 6]. The theory was based on Hamilton's principle and spectral analysis and had led to a series of amplitude dependent mode shapes and resonance frequencies. The purpose of the present work was the examination, by similar methods, of geometrically non-linear vibrations of Bernoulli–Euler beams carrying a finite number of masses at arbitrary positions. A lot of research has been conducted to analyze linear vibrations of beams carrying concentrated masses at arbitrary positions and additional complexities. The present paper is concerned with determination of the geometrically nonlinear frequencies of Bernoulli-Euler beams carrying a finite number of masses at arbitrary positions with general end conditions. Translational and rotational springs are placed at both ends, which allow representing all possible combinations of classical end conditions, as well as elastic supports, by varying the spring stiffness. Therefore, the objective of our study is to present a general solution of the nonlinear problem in order to find the nonlinear frequencies amplitude dependence and make it possible to perform the necessary corrections in rotor dynamics when the linear theory fails in describing appropriately the vibration conditions.

1. Nonlinear formulation

The uniform beam with n masses M_m , shown in figure.1, is made of a material of mass density ρ, Young's modulus E, length L, cross-sectional area S, radius of gyration r and second moment of area of cross section I. $I_m = m_m r_m^2$, I_m is the moment of inertia of the attached mass m_m, where r_m is the radius of gyration with respect to the neutral axis of the beam; k_1 and k_2 are the translational stiffness of the vertical springs supporting the beam at the two ends while k_3 and k_4 are the rotational spring stiffness. Let x be the coordinate along the neutral axis of the beam measured from the right end and $w(x,t)$ be the transverse deflection of the beam, measured from its equilibrium position, and ψ be the slop defined by $\psi = \frac{\partial w(x,t)}{\partial x}$

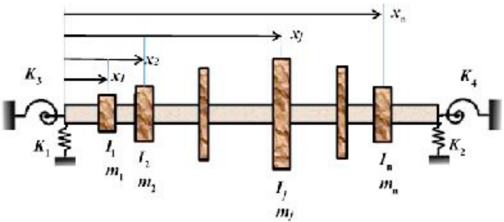

Figure 1 – Restrained beam with N masses

The kinetic energy of the system (beam carrying N masses), denoted in what follows as BCNM, can be expressed as:

$$T = \frac{1}{2}\rho S\int_0^L \left(\frac{\partial w(x,t)}{\partial t}\right)^2 dx + \frac{1}{2}\sum_{j=1}^{n} m_j \left(\frac{\partial w(x_j,t)}{\partial t}\right)^2 + \frac{1}{2}\sum_{j=1}^{n} I_j \left(\frac{\partial \psi(x_j,t)}{\partial t}\right)^2 \tag{1}$$

The BCNM total strain energy can be written as the sum of the strain energy due to the bending denoted as V_{lin}, plus the axial strain energy due to the axial load induced by large deflection denoted as V_{Nlin} [3].

$$V_{Lin} = \frac{1}{2}EI\int_0^L \left(\frac{\partial^2 w(x,t)}{\partial x^2}\right)^2 dx \quad ; \quad V_{Nlin} = \frac{1}{8}\frac{ES}{L}\left[\int_0^L \left(\frac{\partial w(x,t)}{\partial x}\right)^2 dx\right]^2 \tag{2}$$

The transverse displacement function is expanded as a series of basic spatial functions (the linear modes of the BCNM) and the time function is supposed to be harmonic:

$$w(x,t) = q_i(t) w_i(x) = a_i w_i \sin(\omega t) \tag{3}$$