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Analysis of purely harmonic vibrations in non-linear dynamic systems on the example of the non-linear degenerate system

Miroslaw Bocian[a*], Krzysztof Jamroziak[a], Mariusz Kosobudzki[b], Maciej Kulisiewicz[c]

[a]*Wroclaw University of Science and Technology, Faculty of Mechanical Engineering, Department of Mechanics, Materials Science and Engineering, Smoluchowskiego 25, Wroclaw 50-370, Poland*

[b]*General Tadeusz Kosciuszko Military Academy of Land Forces, Czajkowskiego 109, Wroclaw 51-150, Poland*

[c]*Wroclaw University of Science and Technology, Faculty of Technology and Engineering, Armii Krajowej 78, Walbrzych 58-302, Poland*

Abstract

Under loads of dynamic machine components made of composite materials an important role is played by their dissipative-elastic properties. The use of the traditional linear rheological model of the Kelvin type to describe these properties is often an oversimplification. In this paper the authors assume that the non-linear system is asymptotically stable and its response to the harmonic excitation can be described with a periodic function of any form. The authors in this study presented the attempt to apply different concept to the system expressed by the ordinary third-order differential equation, which describes the vibrations of the 1.5 degrees of freedom (i.e. the degenerate system). For example the analyzed system presented in this work illustrates the way of excitation of purely harmonic vibrations as well as the way of the system identification with the use of the energy balance and power balance equations.

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1. Introduction

The search for appropriate models of constitutive relations and on their basis, the development of relevant methods for identifying the dynamic properties of materials is one of the fundamental challenges of modeling. This paper presents the concept of the application of the non-linear dynamic model for this purpose, as shown diagrammatically

* Corresponding author. Tel.: +48-71-320-2765; fax: +48-71-321-1235.
E-mail address: miroslaw.bocian@pwr.edu.pl

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in Fig. 1. This model (Fig. 1a) is a single-mass system and can be used to describe the vibration of a mass m attached by an element made of the tested material to a stationary reference system (Fig. 1b). The mass vibrates in a way, which is primarily determined by the unknown values of the constant parameters k_0, c_0, k, c, h, κ, that describe properties dissipative-elastic properties of the test material.

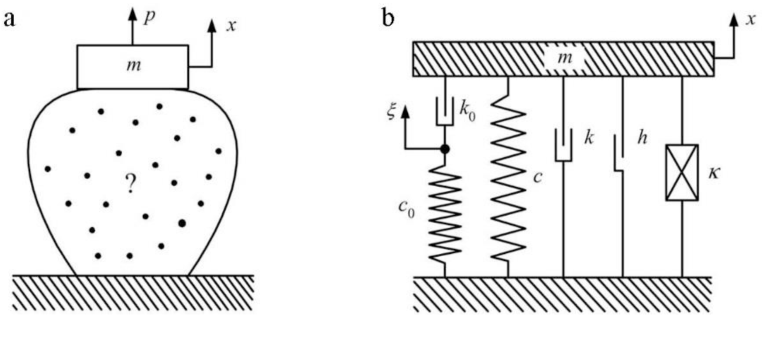

Fig. 1. (a) The scheme of the real system; (b) the scheme of the analyzed system as a model.

Non-linear elements considered in this model include: the element "h" describing the so-called dry friction ($h\cdot$Sign(v)) and the element "κ" describing the so-called mixed member ($\kappa\cdot xv$), where v is the velocity ($v=dx/dt=\dot{x}$). The assumption of the mixed member is justified by the fact that in many materials subjected to tensile (or compression) testing the dependence of the Young's modulus on the deformation velocity is observed [1,2,3]. In the case of the presented model, this module, at the constant velocity $v = v_0$, corresponds to the equivalent stiffness $c_z = c + \kappa v_0$. The proposed model is a dynamic system with 1.5 degrees of freedom. This is the so-called degenerate system [3,4]. Its movement is described by two differential equations in the forms:

$$m\ddot{x} + c_0(x-\xi) + cx + k\dot{x} + h\cdot\mathrm{Sgn}(\dot{x}) + \kappa x\dot{x} = p, \qquad k_0\dot{\xi} = c_0(x-\xi) \tag{1}$$

Having the variable ξ eliminated, the system of equations (1) can be replaced by one third-order equation of the form:

$$\frac{k_0 m}{c_0}\dddot{x} + \left(m + \frac{k_0 k}{c_0}\right)\ddot{x} + \left(k_0 + k + \frac{k_0 c}{c_0}\right)\dot{x} + cx + h\cdot\mathrm{Sgn}(\dot{x}) + \kappa x\dot{x} + \frac{k_0}{c_0}\cdot\frac{d}{dt}\left(h\cdot\mathrm{Sgn}(\dot{x}) + \kappa x\dot{x}\right) = p + \frac{k_0}{c_0}\dot{p} \tag{2}$$

This equation is too complex to have the energy balance method for the identification of unknown constant parameters used directly to it [3]. Therefore, it seems reasonable to control the excitation $p(t)$ so as to obtain the system response in the form of a purely harmonic function. The idea of such proceedings was already presented in earlier works of the authors [3,5]. This paper focuses on its use in the case of the system described by the equation (2). For this purpose it is convenient to know the indicative values of the parameters h and κ as described in the next section of this paper.

2. The method for estimating the values of parameters of non-linear members

The effective excitation in systems of non-linear harmonic vibrations requires approximate knowledge of the values of the parameters occurring with non-linear members. In the case of the adopted model (2) it refers to the parameters h and κ. These parameters can be estimated by using quasi-static tension and/or compression tests carried on samples made from the test material. Such tests necessitate the use of such excitation forces $p(t)$ where the speed of deformation $\dot{x}$ is constant. If $\dot{x} = v_0 = \mathrm{const.}$, the extortion $p(t)$ must meet the equation below in accordance with the relation (2):

$$D + c_z x = p + Bp' \tag{3}$$