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A 2-DOF Model of an Elastic Rocket Structure Under Circulatory Force

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Abstract

It is intended, in this paper, to develop a mathematical and numerical model of an elastic space rocket structure as a Beck's column excited by a follower (or circulatory) force. This force represents the rocket motor thrust that should be always in the direction of the tangent to the structure deformed axis at the base of the vehicle. We present a simplified two degree of freedom rigid bars discrete model. Its system of two second order nonlinear ordinary differential equations of motion are derived via Lagrange's energy method, allowing for a general understanding of the main characteristics of the problem. The proposed equations consider up to third order (cubic) inertia, stiffness and forcing terms. Among other rich nonlinear dynamic behaviour of this model, depending on parameters and initial conditions choices, either stable or unstable limit cycle post-critical steady state solutions are possible. The latter is a form of flutter. Numerical simulations are carried out using Runge-Kutta's 4th order algorithm in Matlab environment.

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1. Introduction

Launcher vehicles (rockets) carry loads from the surface of the Earth to some space mission site. Like any other physical body, is not rigid, so that the structural deformations tend to affect flight dynamics. In this paper, we develop a mathematical model of an elastic space rocket structure as a Beck's column excited by a follower (or circulatory) force. This force is the rocket motor thrust that is always in the direction of the tangent to the structure deformed axis at the base of the vehicle. We present a two degree of freedom rigid bars discrete model. The two second order nonlinear ordinary differential equations of motion are derived via Lagrange's method, allowing for a

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general understanding of the main characteristics of the problem. The equations consider up to third order inertia, stiffness and forcing terms. Depending on parameters and initial conditions choices, either stable or unstable limit cycle post-critical steady-state solutions are possible. The latter is a form of flutter. Flutter first arose in aerospace engineering studying structural breakdowns of aircraft in the years 1910-1930 [1,2,3]. [4] analyzed this problem from the point of view of structural stability. For more adequate analysis, it is necessary to introduce nonlinearities to the mathematical model (up to cubic in our paper), for determination of amplitudes of post-critical stationary states, as in [5,6,7]. Although first studied in aircraft, flutter also occur in missiles, launch vehicles and even suspension bridges (as in the infamous Tacoma Narrows incident). It is a dynamic instability that occurs due to feedback interaction between two or more distinct modes of vibration of a system and introduction of external energy, as per [8, 9, 10]. [11] presents an analytical solution of the fourth order partial differential equation of motion of Beck's column. Flutter causes amplitude oscillations to grow over time causing structural failure.
Numerical simulations are carried out using Runge-Kutta's 4th order algorithm in Matlab environment.

2. Physical model

Fig. 1 is the simplified physical model of the structure of a launcher vehicle. We have two rigid massless bars $\overline{\mathrm{AB}}$ and $\overline{\mathrm{BC}}$, L_1 and L_2 long, pinned at nodes A and B. Displacements are restricted at point A. We have lumped masses $\mathrm{M}_1, \mathrm{M}_2$ and M_3 attached to nodes A, B and C, respectively. Torsional springs k_1 e k_2 provide elastic restoring forces. Viscous dampers of damping constants $\hat{\mu}_1$ and $\hat{\mu}_2$ are added to the joints. We adopt the following modelling hypothesis: 1. $\mathrm{L}_1 = \mathrm{L}_2 = \mathrm{L}$; 2. the bars are rigid and massless; 3. lumped masses $\mathrm{M}_1, \mathrm{M}_2$ and M_3 represent the actual masses of half the bars connected to that point, thus $\mathrm{M}_1 = \mathrm{M}_3 = m$ and $\mathrm{M}_2 = 2m$; 4. the stiffness of the torsional springs represent the elastic properties of the continuous structure, $\mathrm{k}_1 = \mathrm{k}_2 = k$, 5. it is adopted $\hat{\mu}_1 = \hat{\mu}_2 = \hat{\mu}$; 6. the adopted inertial reference is point A, origin of an orthonormal basis $\mathcal{B}_g \equiv (\hat{\imath}, \hat{\jmath})$; 6. motions are restricted to the Axy plan.

The fundamental static equilibrium configuration of the system represents the vehicle at rest in its launch platform. We consider a fixed reference coordinate system fixed at A, as shown in Fig. 1. This system is non-inertial for an observer on Earth, but is assumed to be inertial in the frame of the vehicle on flight. $\vec{\mathrm{F}}$ is a follower (circulatory) non-conservative force applied to C, in the direction of bar $\overline{\mathrm{BC}}$. It models the rocket's thrust force due to combustion gases expansion at the motors in the basis of the vehicle. We do not consider, in this model, its dependence on time.

In a Lagrangian approach, our generalized coordinates are angular displacements θ_1 e θ_2 of bars $\overline{AB}$ and $\overline{BC}$, from their original vertical equilibrium positions. We denote $\theta_1(t) \equiv q_1(t)$ and $\theta_2(t) \equiv q_2(t)$.

3. Mathematical model

3.1 Position vectors of the lumped masses

$$\vec{r}_1 = \vec{0} \tag{1}$$

$$\vec{r}_2 = L\left(\sin q_1 \hat{\imath} - \cos q_1 \hat{\jmath}\right) \tag{2}$$

$$\vec{r}_3 = L\left[\left(\sin q_1 + \sin q_2\right)\hat{\imath} - \left(\cos q_1 + \cos q_2\right)\hat{\jmath}\right] \tag{3}$$