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Procedia Engineering 199 (2017) 589–594

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

Dynamics of two impacting beams with clearance nonlinearity

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Abstract

Analytical solutions describing the transient dynamics of two Euler-Bernoulli beams with tips separated by clearance, are obtained. The tips of the beams impact when one of the beams is harmonically excited. Expressions of transient dynamics are presented as a superposition of particular solutions that satisfy to inhomogeneous boundary conditions, and eigenfunctions series with time dependent coefficients and homogeneous boundary conditions. The transition from impact phase to out-of-contact phase and vice versa is implemented using conditions that switch, involving construction of expressions for shear forces and relative position of beam tips. After each transition from one phase to another, the functions describing the time dependent coefficients in the eigenfunctions series are updated. This update involves the solution of ordinary differential equations with initial conditions corresponding to the end of the previous phase. The system of impacting beams reveals complex dynamics, including chaotic behaviour. Transient dynamics surfaces, time histories of beams deflections, impact forces, coefficients of restitution and phase planes are presented.

Peer-review under responsibility of the organizing committee of EURODYN 2017.

Keywords: Impacting beams; Clearance; Chaos

1. Introduction

Problems related to vibro-impact dynamics of structures are of great interest in mechanical or mechatronic engineering [1, 2]. The dynamics of two impacting beams with clearance nonlinearity can be associated with the study of vibrating bolted and riveted structures with loose fastening, noise generation and chaotic vibrations due to lose connections between structural elements as well as it can be applied to design of impact dampers employing attached flexible beam/beams. In numerous cases non-linear effects caused by clearance may lead to many undesirable effects, in particular to premature failure of structures.

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1877-7058 © 2017 The Authors. Published by Elsevier Ltd.
Peer-review under responsibility of the organizing committee of EURODYN 2017.
10.1016/j.proeng.2017.09.101

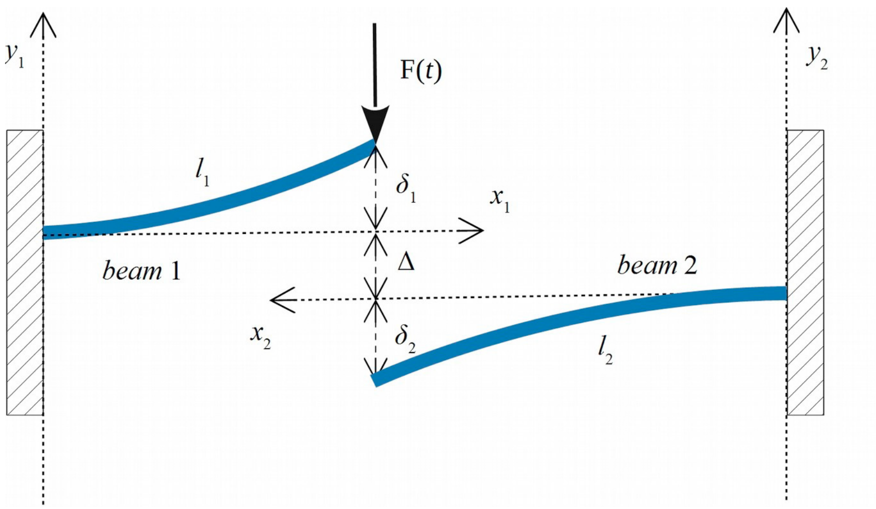

Fig. 1. Impacting beams under harmonic excitation.

2. Mathematical model

The transient dynamics of two impacting cantilever Euler-Bernoulli beams with tips separated by clearance Δ is depicted schematically in Fig. 1. The tips of the beams impact when beam 1 is harmonically excited by a force F(t). l_1 and l_2 are lengths of the beams, and δ_1, δ_2 are the initial uplifts at the free ends. The two beams are not allowed to cross each other at their tips, that is $y_1(l_1,t) \geq y_2(l_2,t)$-Δ always. After the impact-induced phase, they must separate in opposite directions. This interaction of the two beams is governed by the partial differential equations

$$a_1^2 \frac{\partial^4 y_1(x_1,t)}{\partial x_1^4} + \frac{\partial^2 y_1(x_1,t)}{\partial t^2} = 0 \quad , \quad a_2^2 \frac{\partial^4 y_2(x_2,t)}{\partial x_2^4} + \frac{\partial^2 y_2(x_2,t)}{\partial t^2} = 0 \tag{1}$$

with different set of boundary conditions for the impact phase and for the out-of-contact phase. For the impact phase the boundary conditions are

$$y_1(0,t)=0 \quad , \quad y_2(0,t)=0 \quad , \quad \frac{\partial y_1(0,t)}{\partial x_1}=0 \quad , \quad \frac{\partial y_2(0,t)}{\partial x_2}=0 \quad , \tag{2}$$

$$M_1(l_1,t)=E_1 I_1 \frac{\partial^2 y_1(l_1,t)}{\partial x_1^2}=0 \quad , \quad M_2(l_2,t)=E_2 I_2 \frac{\partial^2 y_2(l_2,t)}{\partial x_2^2}=0 \quad , \tag{3}$$

$$Q_1(l_1,t)=E_1 I_1 \frac{\partial^3 y_1(l_1,t)}{\partial x_1^3} \quad = \quad Q_2(l_2,t)+F(t)=E_2 I_2 \frac{\partial^3 y_2(l_2,t)}{\partial x_2^3}+F(t) \quad , \tag{4}$$

$$y_1(l_1,t)=y_2(l_2,t)-\Delta \quad . \tag{5}$$

Here E_i ,I_i,ρ_i,A_i, are the Young modulus, area moment of inertia, mass density and cross-section of i-th beam, $a_i=\sqrt{E_i I_i / \rho_i A_i}$, i=1,2. Boundary conditions (2), (3) are identical for the impact and out-of-contact phases. Conditions (2) correspond to fixed both the position and slope of the cantilever beams at points x_1=0 and x_2=0. Since no external bending moments are applied at the beam tips $x_1=l_1$ and $x_2=l_2$, the bending moments $M_1(x_1,t)$ and $M_2(x_2,t)$ at that locations are zero (conditions (3)). The shearing force $Q_1(x_1,t)$ at the tip $x_1=l_1$ is equal to the sum of the applied force $F(t)$ and the reaction force $Q_2(x_2,t)$ at the tip $x_2=l_2$ (condition (4)). Condition (5) is the necessary