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A Piecewise-Linear Model of Intracranial Pressure Dynamics

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Abstract

Elevated intracranial pressure (ICP) is an extremely dangerous condition for patients suffering from traumatic brain injury, hydrocephalus, or related neurological disorders. To make informed decisions when treating such patients, clinicians must understand how the body controls ICP by regulating the rates at which cerebrospinal fluid (CSF) is formed and reabsorbed. Mathematical models can aid in this task. Of particular interest are models that can help us understand the transition between the stable, approximately constant values of ICP found in healthy individuals, and pathological oscillatory behaviors such as those observed with *plateau waves*, in which ICP exhibits steady oscillations between high and low pressures. In this paper, we develop a mathematical model of ICP dynamics with the goal of illustrating how the transition to oscillatory ICP dynamics can arise from processes that regulate CSF formation. A simple low-dimensional model is built that couples brain tissue mechanics and CSF hydraulics. Balance of mass and linear momentum result in a damped linear oscillator in terms of a single configuration variable representing deformation of brain tissue caused by changes in ICP. We focus on the case where the CSF supply rate is regulated by a piecewise mechanism based on the total volume of CSF. We show that the resulting piecewise-linear dynamical system can exhibit limit cycles in a manner consistent with plateau waves. We conclude by physically interpreting our results and discussing their potential clinical implications.

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Keywords: brain, intracranial pressure, piecewise model, limit cycles

1. Introduction

Permanent brain damage is a possible outcome when patients suffer from elevated intracranial pressure (ICP), as the high pressure slows the supply of blood, and therefore oxygen, to the brain [3]. Therefore, it is essential for clinicians to understand how the body naturally regulates ICP. Of particular medical interest are transitions between healthy and unhealthy levels of ICP, as well as the transition to a pathological oscillatory state, known as plateau waves, where ICP fluctuates between normal and dangerously-high values with a period of about 30 minutes. In this paper, we use the balance of mass and balance of linear momentum to develop a simple mathematical model of ICP dynamics that incorporates aspects of brain tissue mechanics with the hydraulics of the cerebrospinal fluid (CSF). We impose a piecewise control mechanism on the formation rate of CSF based on the total CSF volume, and show that the resulting model can exhibit a transition to oscillatory solutions in a manner consistent with plateau waves.

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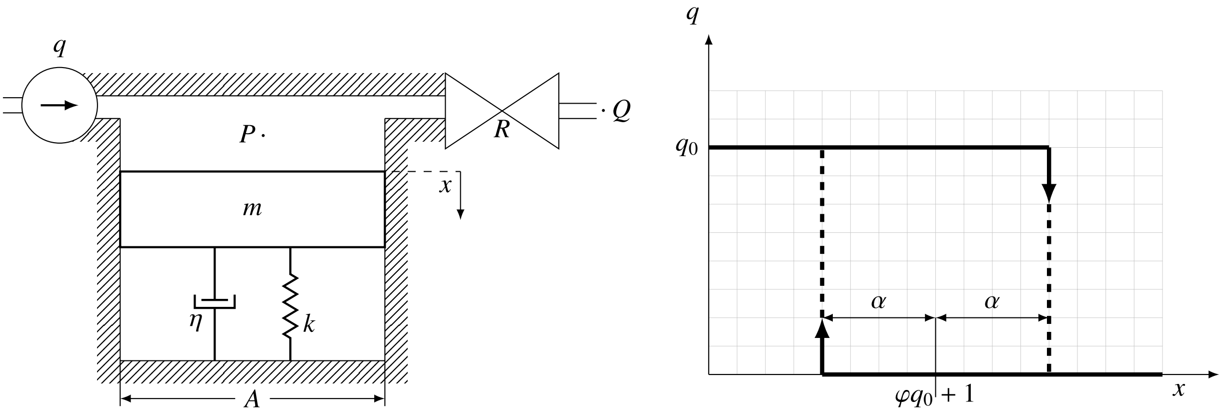

Fig. 1: Mechano-hydraulic model of the ventricular CSF system. (*left*) A single reservoir model with m the effective mass of the ventricle wall, x the effective deformation of the brain tissue, and k and η its effective stiffness and damping constants. The intracranial pressure is P, Q is the venous blood pressure, q is the volumetric rate of CSF formation, and A is the ventricle wall surface area. The effective spring is unstretched when $x = 0$. (*right*) The CSF formation rate q is allowed to vary hysteretically with the deformation, x (see Eq. 6). If x takes values in the overlap region, i.e. if $-\alpha < x - (\varphi q_0 + 1) < \alpha$, then q retains its previous value. The parameters shown in the right figure are for the dimensionless system (Eqs. 5 and 6).

2. A Piecewise Mechano-Hydraulic Model

We are interested in studying the transition from a constant steady state of ICP to oscillatory plateau waves, corresponding to a bifurcation to stable limit cycles in the system [1,6]. Effective mathematical models are always abbreviated descriptions of reality [4], and here our interest is in studying fundamental dynamical phenomena related to ICP without unnecessary detail. In particular, since plateau waves are primarily temporal phenomena, we do not account for spatial variations in either ICP or the brain tissue deformations in response to it. Instead, we construct our model by idealizing the intracranial geometry to include a single CSF reservoir and a single deformational degree of freedom for the brain tissue. An illustration of this idealization is shown in Fig. 1 (left).

Cerebrospinal fluid (CSF) is a clear, colorless fluid that is stored mainly in the brain's ventricles [3], is formed by the choroid plexus at the rate of a few milliliters per day, and is eventually absorbed back into the blood stream. We can quantify the relationship between CSF formation, storage, and absorption through the balance of mass principle. Since CSF is mostly water, it is reasonable to approximate it as an incompressible fluid. We assume a volume rate of absorption that is proportional to the difference between the intracranial pressure, P, and the venous blood pressure, Q (Fig. 1, left). We write the proportionality constant $1/R$, as in the electric circuit analogies of Marmarou [2] and Ursino [5]. Balancing mass on the ventricle, we have

$$q - \frac{P - Q}{R} = \dot{V}\,, \tag{1}$$

where q is the volume rate of CSF formation, V is the total volume of ventricular CSF, and the overdot denotes the derivative with respect to time. The intracranial pressure also affects how the surrounding brain tissue responds to changes in shape. We consider brain tissue deformation to be adequately captured by a single degree of freedom, x, and model the ventricle as having an approximately constant surface area A so that the CSF volume is $V = V_0 + Ax$, where V_0 is the undeformed ventricle volume. Thus, $x = 0$ in the undeformed configuration. We further treat the surrounding tissue as a linear viscoelastic, Kelvin-Voigt solid. For our single degree of freedom system, this corresponds to the addition of a linear spring in parallel with a dashpot, as shown in Fig. 1 (left). The tissue that is being displaced has a mass m, representing the *effective* mass of the parts of the brain affected by changes in CSF volume.

Linear momentum balance applied to the effective mass then yields

$$m\ddot{x} + \eta\dot{x} + kx = PA\,, \tag{2}$$

where PA is the force generated by the intracranial pressure acting over the ventricle surface area. Since $\dot{V} = A\dot{x}$, the two governing equations (Eqs. 1 and 2) are coupled through the intracranial pressure P and deformation rate $\dot{x}$.