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Procedia Engineering 199 (2017) 607–612

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

Estimation problems for uncertain nonlinear dynamical systems with ellipsoidal state constraints

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Abstract

The paper deals with the estimation problems for uncertain nonlinear dynamical systems in the case when a probabilistic description of noise and errors is not available, but only bounds on them known. Such models are found in many applied areas ranged from engineering problems from physics to economics as well as to biological and ecological modeling when it occurs that a stochastic nature of the errors is questionable because of limited data or because of complexity and nonlinearity of the model. As alternative to a stochastic characterization, a so-called set-membership approach has been proposed and intensively developed in the last decades. The solution of many control and estimation problems under uncertainty involves constructing reachable sets and their analogs. For models with linear dynamics under such set-membership uncertainty, there are several constructive approaches, which allow finding effective estimates of reachable sets. Certainly, concrete problems are mostly nonlinear in their parameters and the set of feasible system states is usually non-convex or even non-connected. The key issue in nonlinear set-membership estimation is to find suitable techniques, which produce related bounds for the set of unknown system states without being too computationally demanding. Here the problem of estimating reachable sets of nonlinear dynamical control systems with combined bilinear and quadratic nonlinearity and with uncertainty in initial states is studied. Applying results of the theory of trajectory tubes of control systems and related techniques of differential inclusions theory we present approaches that allow finding the upper ellipsoidal estimates of reachable sets. The main new result consists in obtaining ellipsoidal estimates for reachable sets of nonlinear dynamical system with additional state constraint of ellipsoidal type. Related numerical algorithms, examples and results of computer simulations are included.

Peer-review under responsibility of the organizing committee of EURODYN 2017.

Keywords: Nonlinear control systems; estimation under uncertainty; reachable sets; ellipsoidal calculus

1. Introduction

The main objective of the paper is to study the estimation problems for uncertain systems in the case when a probabilistic description of noise and errors is not available, but only bounds on them are given [1–11].

The solution of many control and estimation problems for dynamical systems operating under uncertainty involves constructing related reachable sets and their analogs. For models with linear dynamics under such set-membership

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1877-7058 © 2017 The Authors. Published by Elsevier Ltd.
Peer-review under responsibility of the organizing committee of EURODYN 2017.
10.1016/j.proeng.2017.09.104

uncertainty there are several constructive approaches which allow finding effective estimates of reachable sets, one of them is based on the ellipsoidal calculus [2,3,5,6]. However concrete applied problems are mostly nonlinear and the set of feasible system states is usually non-convex or even non-connected. In this paper the modified state estimation approaches which use the special structure of nonlinearity and uncertainty are presented. The study of reachable sets for the class of systems considered here is not only of theoretical interest, the results may also find use in many applied models whose dynamic characteristics contain similar nonlinearities, e.g. [12–14].

We present here new results of guaranteed estimation of the states of dynamical systems under conditions of uncertainty and nonlinearity based on previously developed techniques [2,3,5–7], but we consider here new more complicated cases of dynamics combining simultaneously bilinear and quadratic state velocities of dynamical systems. This novelty of the problem setting requires a more refined analysis of the problem data, which is the subject of the paper. We also study the estimation problems for such systems under a additional restriction on state positions defined by a given ellipsoid. Therefore, the objective of this paper is to present constructive approaches which allow finding effective estimates of reachable sets of control systems with nonlinear dynamics and with uncertainty of set-membership kind.

2. Problem formulation

In this section, we introduce necessary notations and give the basic formulation of the problem.

2.1. Basic notations and preliminaries

Let us introduce the following basic notation. Let R^n be the n-dimensional vector space, $\operatorname{comp} R^n$ be the set of all compact subsets of R^n, $R^{n\times m}$ stands for the set of all real $n \times m$-matrices, $x'y = (x, y) = \sum_{i=1}^{n} x_i y_i$ be the usual inner product of $x, y \in R^n$ with prime as a transpose,

$$\|x\| = \|x\|_2 = (x'x)^{1/2}, \quad \|x\|_\infty = \max_{1\le i\le n} |x_i|$$

be vector norms for $x \in R^n$, $I \in R^{n\times n}$ be the identity matrix, $\operatorname{tr}(A)$ be the trace of $n\times n$-matrix A (the sum of its diagonal elements). We denote by $B(a, r) = \{x \in R^n : \|x - a\| \le r\}$ the ball in R^n with a center $a \in R^n$ and a radius $r > 0$ and by

$$E(a, Q) = \{x \in R^n : \ (Q^{-1}(x - a), (x - a)) \le 1\}$$

the ellipsoid in R^n with a center $a \in R^n$ and with a symmetric positive definite $n \times n$-matrix Q.

Consider the following system

$$\dot{x} = A(t)x + f(x)d + u(t), \quad x_0 \in \mathcal{X}_0, \quad t \in [t_0, T], \tag{1}$$

where $x, d \in R^n$, $\|x\| \le K$ $(K > 0)$, $f(x)$ is the nonlinear function, which is quadratic in x, $f(x) = x'Bx$, with a given symmetric and positive definite $n \times n$-matrix B.

Control functions $u(t)$ in (1) are assumed to be Lebesgue measurable on $[t_0, T]$ and satisfying the constraint $u(t) \in \mathcal{U}$ for a.e. $t \in [t_0, T]$ (here $\mathcal{U}$ is a given set, $\mathcal{U} \in \operatorname{comp} R^n$). The $n \times n$-matrix function $A(t)$ in (1) has the form

$$A(t) = A^0 + A^1(t), \tag{2}$$

where the $n \times n$-matrix A^0 is given and the measurable $n \times n$-matrix $A^1(t)$ is unknown but bounded, $A^1(t) \in \mathcal{A}^1$ $(t \in [t_0, T])$,

$$A(t) \in \mathcal{A} = A^0 + \mathcal{A}^1, \ \ \mathcal{A}^1 = \{A=\{a_{ij}\}\in R^{n\times n} : |a_{ij}|\le c_{ij}, \ i, j=1, \dots n\}, \tag{3}$$

where $c_{ij} \ge 0$ $(i, j = 1, \dots n)$ are given.

We will assume that $\mathcal{X}_0$ in (1) is an ellipsoid, $\mathcal{X}_0 = E(a_0, Q_0)$, with a symmetric and positive definite matrix $Q_0 \in R^{n\times n}$ and with a center a_0.