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Non-holonomic dynamics of a ball moving inside a spherical cavity

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Abstract

The ball type vibration absorbers are popular to be used at slender structures like masts or towers when a vertical space available and maintenance possibilities are limited. In general, the absorber consists of a cavity in which a heavy ball is rolling. In the paper, the response of a heavy ball rolling inside a semi-spherical cavity under horizontal kinematic excitation is investigated. The system with six degrees of freedom with three non-holonomic constraints is considered. The contact between the ball and the cavity surface is supposed to be perfect without any sliding. The system is strongly non-linear and, consequently, the full 3D mathematical model should be carried out and discussed. With respect to previous experiences the basic strategy declines the Lagrangian procedure including conventional constraints via Lagrangian multipliers. Instead of that the Gibbs-Appell formulation is used. The system has an auto-parametric character. The most important post-critical regimes are outlined and qualitatively evaluated in resonance domain. Numerical experiments have been performed when excitation frequency is slowly swept up and down. Some applications in civil engineering as a tuned mass damper used on slender structures is outlined and compared with a conventional pendulum damper. Strengths and weaknesses of both absorbers types are discussed.

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Keywords: tuned mass damper; moving ball; Appell-Gibbs formulation; non-holonomic systems; auto-parametric systems

1. Introduction

Various types of passive vibration absorbers are often used in civil engineering for suppression of wind induced vibration. Usual pendulum-style passive absorbers, see, e.g., [1], utilize the auto-parametric resonance for their damping effect. Although they are very effective and reliable several disadvantages limit their application. Dimensions of the pendulum and namely its suspension length cannot be neglected or minimized and sometimes the structure cannot accommodate this device. This is particularly true for existing structures, where an absorber should be installed as a supplementary equipment. Also horizontal constructions, like foot bridges, could hardly include a pendulum-style absorber. Moreover, the complete installation has to remain accessible to allow a regular maintenance.

The ball-type absorber represents an alternative, which is less spatially-demanding and practically maintenance-free. The basic principle comes out of a rolling movement of a metallic ball of a radius r inside of a metallic rubber coated spherical cavity of a radius $R > r$, Fig. 1a. The system can be closed in an airtight case. Its vertical dimension

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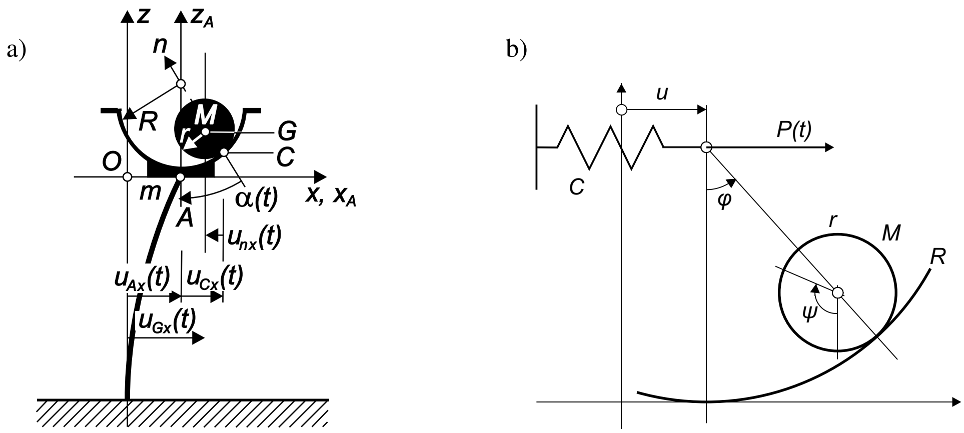

Fig. 1. a) Outline of the ball absorber. b) Scheme of the simplified 2D model.

depends only on the dimension of the rolling ball and thus can be relatively very small. Such device can be used in cases where a pendulum absorber is inapplicable due to lack of vertical space or difficult maintenance.

First papers dealing with the theoretical and practical aspects of ball absorbers have been published during the last two decades, see [2] and [3]. The first analysis of the problem on the basis of the rational dynamics has been published by the authors in [4] and later extended in [5]. The approach in the referenced papers was based on the basic planar mathematical model, Fig. 1b. The theoretical derivation together with its numerical evaluation was compared to practical application up to the state of the realization including some results of long-term in situ measurements.

Authors tried in the past to formulate this problem by a classical way constructing the Hamiltonian functional with non-holonomic constraints and the respective Lagrangian governing system for the planar and space problem. The planar configuration leads to the quite transparent system, see [5], which can be treated analytically. However, the three-dimensional approach provides the differential system which is too complicated and its physical interpretation can be multivalent. The system should be treated almost only numerically. Therefore, it is not very suitable for further discussion. For easier analysis of the dynamic system the problem is formulated using Appell-Gibbs function, namely when the system includes non-holonomic constraints. In general, the Appell-Gibbs approach follows from the Gaussian fifth form of the basic principle of dynamics. A similar problem related with anti-seismic protection of a building has been discussed for instance in [6] or [7] using also Appell-Gibbs procedure. The main advantage of the Appell-Gibbs function consists in easier problem definition and easier more transparent introduction of non-holonomic constraints.

The basis and theoretical background of the Appell-Gibbs approach can be found in monographs, e.g., [8] or [9]. It is worthy to refer also additional papers by [10] or [11] presenting rigorous and well conducted analytical basis. Formulation of the Appell-Gibbs environment with a significant generalization of non-holonomic constraints has been discussed by Udwadia [12] and others. The high order systems investigated by Appell-Gibbs approach can be found in [13] and subsequent papers of these authors. Non-conventional procedure dealing with time variable mass has been presented by Yong-fen [14]. It is worth noting that the Appell-Gibbs approach is noticeable also in other disciplines as for instance theoretical physics studying movement and interaction of particles.

2. Mathematical model

The traditional approach which is currently used in the engineering practice is based on a simplified planar model. The dynamic character of the complete structure is represented by a linear SDOF system (mass M_s and stiffness) where the total mas M_s includes the structure, cavity and the ball. The ball with mass M is moving in the cavity freely in a vertical plane only, as it is outlined in Fig. 1b. This procedure results in a 2DOF system. In [5] authors presented a thorough analysis of an auto-parametric non-linear planar model, describing interaction of the ball absorber and the structure. Its auto-parametric character originated from coupling between the ball, cavity and elastic structure. The