Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 196 (2017) 714 – 722

Procedia Engineering

www.elsevier.com/locate/procedia

Creative Construction Conference 2017, CCC 2017, 19-22 June 2017

Precedence pattern permutations creating criticality constellations: Exploring a conjecture on non-linear activities with continuous links

Gunnar Lucko[a]*, Yi Su[a]

[a] *Department of Civil Engineering, Catholic University of America, 620 Michigan Avenue NE, Washington, DC 20064, USA*

Abstract

The inaugural challenge of the 2016 Creative Construction Conference has posed two related questions on how many possible criticality constellations with different behavior for delays and acceleration exist and how said constellations can occur for non-linearly and monotonously progressing activities that have continuous relations. This paper systematically solves these questions by performing a thorough literature review, assembling theoretical foundations for link constellations, performing a computer simulation of all possible permutations, and providing a mathematical proof by contradiction. It is found that (for the initially assumed self-contained activities in a network schedule that exhibit only a linearly growing production) the three newly hypothesized criticality constellations cannot exist. Non-linear activity constellations with diverging or converging relative productivities are examined next. Lags in networks become buffers in linear schedules. It is found that a non-linear curvature of the progress may induce middle-to-middle relations besides those between starts and finishes. If multiple curvatures are allowed, then partial segments can form relations, which increases the number of criticality constellations.

Peer-review under responsibility of the scientific committee of the Creative Construction Conference 2017

Keywords: Precedence Diagramming; link constellations; continuous precedence relations; classification of critical activities

1. Introduction

The theory of network schedules in construction project management is celebrating its 60th anniversary after Kelley and Walker [1] conceived the widely known critical path method around December 1956 [2] for schedules that are represented as networks. It constitutes a simplified form of linear programming with a rigorous equation structure of start plus duration equals finish (S + D = F) for all activities. It has two 'passes', first adding all activity durations sequentially from the origin and taking the maximum at any merge of several predecessors under an 'as-

* Corresponding author. Tel.: +1-202-319-4381; fax: +1-202-319-6677.
E-mail address: lucko@cua.edu.

1877-7058
Peer-review under responsibility of the scientific committee of the Creative Construction Conference 2017
doi:10.1016/j.proeng.2017.07.238

early-as-possible' assumption (forward pass, which also gives the project duration); second subtracting all activity durations sequentially from the terminus and taking the minimum at any merge of several successors under an 'as-late-as-possible' assumption (backward pass). If links carry lead or lag durations, they are added or subtracted analogously. Comparing the earliest and latest starts and finishes from these passes determines the flexibility (float) of activities to be delayed without harm. Like activities and leads or lags, float is measured in time units, commonly workdays. If it is zero, the activity is deemed critical. This means that its delay will immediately impact the project duration. Of interest for this paper is how the link types in the sequence impact the possible criticality of an activity.

2. Literature review

Recent papers by Hajdu [e.g. 3] have renewed a focus on development and theory of the precedence diagramming method by Fondahl [4] as expanded by IBM [5] to four different link types, as opposed to the default finish-to-start link that had limited the realism of Kelley and Walker's [1] critical path method. Continuous relations between two activities with repetitive tasks were shown as a multitude of task links [3]. The transition from end-point-links to true continuous relations between activity pairs was demonstrated for monotonous and invertible functions to model non-linear progress of activities [6], e.g. increasing or decreasing productivities that are caused by a changing numerator or denominator, e.g. learning or fatigue or a changing geometry of the work product itself, e.g. a deepening trench.

Theoretical studies of network scheduling have punctuated its history, which this paper can only review briefly. Roy [7] named the Metra potential method after his consulting firm. Unknown to Fondahl [4], it pioneered activity-on-node diagrams instead of Kelley and Walker's [1] activity-on-arrow diagrams, and enabled links between activity starts. Confusingly, the name of non-time-scaled network diagrams has become intermingled with another technique, the program evaluation and review technique [8], which introduced a three-point estimate of probabilistic durations. Interestingly, the literature continued to mention the complete four different link types (see following section) only after scheduling calculations were explained [9], which fittingly echoed their historically 'later' addition to theory.

Besides describing antecedents of linear schedules, which have been reviewed by Gattei and Lucko [10], Rösch [11] compared Roy's [7] and Fondahl's [4] methods with small examples in bar chart, network schedule, and linear schedule representations, including some older variants. Wiest [12] made a seminal contribution by describing how activity pairs can be linked in a manner that impacts the project duration in a normal, reverse, neutral, or perverse manner. Kallantzis and Lambropoulos [13] showed how such reverse behavior is explained with a linear schedule.

Hajdu [14], in a review of precedence diagramming for which these authors provided sources from the literature, highlighted studies that recently have sought to generalize end-point-links to newly being able to attach anytime during mid-activity: The chronographic method [e.g. 15] and graphical diagramming method [16] for time-scaled precedence diagrams [17] (called line schedules for short), which unlike bar charts hold multiple activities per row and may have evolved from the arrow diagramming method; the 'bee-line' diagram [18] that also allows multiple links between point pairs on two activities; and the relationship diagramming method [19], whose single link carries some explanatory codes. Yet these three methods are beyond the scope of this paper and are left for future research.

2.1. Network schedule assumptions

Assumptions for this paper have been set by the challenge [20, 21] as representing schedules as activity-on-node networks, i.e. acyclic (no loops) graphs, which are directional from an origin to a terminus node, whose activities are connected at their ends via discrete end-point-links and progress at a constant productivity without interruptability.

Assuming that two activities are connected via only one link and that it may attach at either the start or finish of a predecessor and successor, $2 \times 2 = 4$ possible one-link relations can exist: Finish-to-start, start-to-start, finish-to-finish, and start-to-finish. Broadening this common list to connect the four start and finish points of the activity pair with more links gives two-link, three-link, and of four-link end-point-relations (the upper limit) as follows. Each link can carry a lead or lag, which means either a period seen from predecessor or successor view [22] or a negative or positive link duration [23]. Together 15 different permutations of connecting the activity pair can exist as follows:

- One-link permutations: {FS}; {SS}; {FF}; {SF}
- Two-link permutations: {FS, SS}; {FS, FF}; {FS, SF}; {SS, FF}; {SS, SF}; {FF, SF}
- Three-link permutations: {FS, SS, FF}; {FS, SS, SF}; {FS, FF, SF}; {SS, FF, SF}
- Four-link permutations: {FS, SS, FF, SF}