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Nonlinear Analytical Model of Composite Concrete Slab Free and Forced Vibrations

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Abstract

A composite concrete slab consists of two layers of concrete cast at different times: a bottom layer which is a prefabricated element and a top layer, which is concrete cast on the site. There is also interface between the bottom and top layer of the slab. The paper describes the results of calculations of a nonlinear composite slab model. A numerical model was first constructed, which takes into account the existence of friction stress in the contact between the layers. A substitute slab model, as a system with the single degree of freedom, was then created based on the results of numerical calculations. The existence of friction in the model is responsible for nonlinear phenomena – the rigidity of the model is not constant and depends on the model's deflection. An analytical equation of the model was defined based on the above assumption. It was shown that the composite concrete slab is of a soft characteristic, which can be described by the Duffing equation. A closed-form solution was obtained by making the above assumption. The free response of the nonlinear system, as well as the response of the system to a harmonic excitation were discussed. The response of the nonlinear model differs significantly from the linear model's response. For example, bifurcation appears with some bands of the harmonic load. Moreover, the natural frequency of the model is not constant. The frequency value is decreasing as the amplitude of vibrations is increasing. The observations described may be used for determining composite concrete slabs' condition.

Keywords: composite concrete slab; nonlinear model; vibrations

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1. Introduction

Composite concrete slabs consist of two layers of concrete: a bottom layer which is a prefabricated element and a top layer which is a concrete cast on site [1]. Vertical reinforcement is provided along the interface (joint) between the two layers. Experimental investigations [2] have shown that concrete in the interface area is not homogenous and local, intensive deformations of the bottom and top layer in the interface area occur after causing a certain value of shear stresses. Very small relative slipping of the concrete layers occurs in the interface as a result. Such displacements don't cause the loss of bearing capacity by the slab, however, the rigidity of the structure is slightly decreasing as a result of such displacements. Such a decrease can be observed as a change of frequency of the slab's free vibrations. The paper defines a friction model in the joint and a nonlinear model of a construction with a single degree of freedom. The model's free vibrations and vibrations excited with harmonic force are then analysed. The outcome of the analyses performed is a proposal to determine the condition of the joint through observation of vibrations. There is also material damping in the slab's layer, which is modeled as a viscous friction.

A interface model was adopted in which the shear stress ν existing in the interface meet the condition:

$$|\nu| \leq \nu_{\mathrm{fr}}, \tag{1}$$

where ν is shear stress. If $|\nu| < \nu_{\mathrm{fr}}$, than the mutual displacement w of layers is not taking place, hence the velocity $\dot{w}$ of such displacement equals zero (Fig. 1a). If $|\nu| = \nu_{\mathrm{fr}}$, than the displacement of the top layer in relation to the bottom layer occurs, hence the velocity $\dot{w}$ is different than zero (Fig. 1b). The sense of stress ν_{fr} is opposite to the sense of velocity $\dot{w}$ (Fig. 1c). It should be noticed, which is very important for further considerations, that the occurrence of displacements in the interface is accompanied by the change of rigidity of the whole construction.

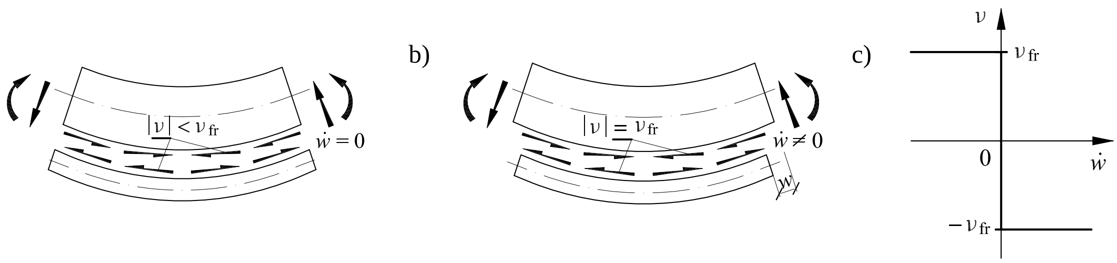

Fig. 1. Friction stresses in the interface: a) $|\nu| < \nu_{\mathrm{fr}}$, when $\dot{w} = 0$, b) $|\nu| = \nu_{\mathrm{fr}}$, when $\dot{w} \neq 0$, c) dependency $\nu(\dot{w})$

2. Slab model equation

The considerations are limited to a freely supported composite slab consisting of two layers. The rigidity of the slab model depends on shear stress (ν) in the interface. If the absolute value of such stress is smaller than ν_{fr}, the model rigidity corresponds to the rigidity of a monolithic slab. After inducing shear stress with the value $|\nu| = \nu_{\mathrm{fr}}$ on the section l_{v}, a shift occurs in the interface and rigidity is decreasing. The rigidity of the slab model hence depends on its deflection value.

The considerations are limited to the first form of model vibrations, i.e. to the bending line caused by the uniform distributed load q (Fig. 2a). The load at which the ν_{fr} stress occurs in the interface is designated as q_{fr}. The following can be stated by using the dependency for shear stress:

$$q_{\mathrm{fr}} = \frac{2Ib\,\nu_{\mathrm{fr}}}{S_{\mathrm{t}}l}, \tag{2}$$

where: b – width of cross section, S_{t} – static moment of part of the cross section situated below the interface, calculated relative to the neutral axis, I – moment of inertia of the cross section, l – length of the slab. If $q > q_{\mathrm{fr}}$, then the stress $\nu = \nu_{\mathrm{fr}}$ exists in the joint at the length of: