Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 194 (2017) 407 – 413

Procedia Engineering

www.elsevier.com/locate/procedia

10th International Conference on Marine Technology, MARTEC 2016

Similarity Solution of Unsteady MHD Boundary Layer Flow and Heat Transfer past a Moving Wedge in a Nanofluid using the Buongiorno Model

M. Ali[a,*], M. A. Alim[b], R. Nasrin[b], M. S. Alam[a], M. J. Haque Munshi[c]

[a]*Chittagong University of Engineering & Technology, Department of Mathematics, Chittagong -4349, Bangladesh*
[b]*Bangladesh University of Engineering and Technology, Department of Mathematics, Dhaka -1000, Bangladesh*
[c]*Hamdard University Bangladesh, Department of Mathematics, Munshigonj, Bangladesh*

Abstract

The present work is focused on the unsteady MHD boundary layer flow and heat transfer over a wedge stretching surface moving in a nanofluid with the effects of various dimensionless parameters by using the Boungiorno model. The solution for the velocity, temperature and nanoparticle concentration depends on parameters like Prandtl number Pr, Brownian motion Nb, thermophoresis Nt, unsteadiness parameter A, velocity ratio parameter λ, pressure gradient parameter β and magnetic parameter M. The local similarity transformation is used to convert the governing partial differential equations into coupled higher order non-linear ordinary differential equations. These equations are numerically solved by using fourth order RungeKutta method along with shooting technique. Numerical results are obtained for distributions of velocity, temperature and nanoparticle concentration, as well as, for the skin friction, local Nusselt number and local Sherwood number for several values of governing parameters. The results are shown in graphically and as well as in a tabular form. From the graph the results indicate that the velocity increases for increasing values of magnetic parameter, unsteadiness parameter and pressure gradient parameter but decreases for velocity ratio parameter. The temperature profile increases for thermophoresis and Brownian motion parameter but reverse results arises for Prandtl number and velocity ratio parameter. On the other hand, nanoparticle concentration decreases for thermophoresis parameter, Lewis number and velocity ratio parameter. But in case of Brownian motion parameter the concentration decreases up to $\eta < 1$ and then increases. Besides, the present results are compared with previously published work and found to be in good agreement.

Peer-review under responsibility of the organizing committee of the 10th International Conference on Marine Technology.

Keywords: MHD; nanofluid; stretching; wedge flow; unsteady

1. Introduction

In 1904, Ludwig Prandtl developed a boundary layer theory to understand the flow of a viscous fluid near a solid surface. Later, in 1931 on the basis of this boundary layer theory, V. M. Falkner and S. W. Skan developed a theory which is not parallel to the direction of flow called wedge flow. So due to the enormous application of this theory in the real world problem a lot of work has been done over the last years. Among them are, Seddeek et al. [1] has found the similarity solutions for a steady Falkner- Skan flow and heat transfer over a wedge with variable viscosity and thermal conductivity, Michael [2] analysed the Falkner- Skan flow over a wedge with slip boundary conditions, Yacob

* Corresponding author. Tel.:+8801713109929
E-mail address: ali.mehidi93@gmail.com

Peer-review under responsibility of the organizing committee of the 10th International Conference on Marine Technology.
doi:10.1016/j.proeng.2017.08.164

et al. [3] has studied the Falker- Skan flow for a static or moving wedge in a nanofluids, Hayat et al. [4] discussed the Falkner- Skan flow in a power law fluid with mixed convection and porous medium, Prasad et al. [5] analysed the MHD mixed convection flow over a permeable non-isothermal wedge and Ashwini et al. [6] discussed the unsteady MHD accelerating flow past a wedge with thermal radiation and internal heat generation or absorption. Again the common heat transfer fluids like water, ethylene glycol, and engine oil have limited heat transfer capabilities due to their low heat transfer properties. But metals have higher thermal conductivities than these base fluids. Therefore, if we combine the two substances to produce a heat transfer medium that behaves like a fluid but has the higher heat transfer properties. This fluids are known as nanofluid which was first introduced by Choi in 1995 in order to develop advanced heat transfer fluids with substantially higher conductivities. Therefore nanofluids are suspensions of metallic (such as Cu, Al, Fe, Hg, Ti etc) or non-metallic (like Al_2O_3, CuO, SiO_2, TiO_2) nano-powders in base fluid (water, engine oil, ethylene glycol etc). In 2006, Buongiorno explained convective transport in nanofluids who considered seven slip mechanisms that can produce a relative velocity between nanoparticles and the base fluid. Among these, only Brownian diffusion and thermophoresis were found to be important. Later, Nield [7] and Kuznetsov [8] investigated the natural convective boundary layer flow of a nanofluid by using Buongiorno model. By applying the model of Nield [7], Khan and Pop [9] first studied the boundary layer flow of nanofluid past a linearly stretching sheet. Mutuku [10] discussed MHD nanofluid flow over a permeable vertical plate with convective heating, Kandasamy et al. [11] investigated the MHD boundary layer flow of a nanofluid past a vertical stretching permeable surface with suction/injection. Mustafa et al. [12] analysed the flow of a nanofluid near a stagnation point towards a stretching surface. Rana and Bhargava [13] has shown the steady, laminar boundary layer flow due to the nonlinear stretching flat surface in a nanofluid. Later, Makinde et al. [14] discussed the combined effects of buoyancy force and magnetic field on stagnation-point flow and heat transfer in a nanofluid flow towards a stretching sheet. A theoretical study of unsteady boundary layer flow of a nanofluid over a permeable stretching/shrinking sheet was reported by Bachok et al. [15]. Motivated by the work done in mentioned above, the present paper studied the Falkner- Skan boundary-layer problem for a moving wedge immersed in a nanofluid in presence of magnetic field. Using similarity transformations, the governing partial differential equations are reduced to a set of coupled nonlinear ordinary differential equations with corresponding boundary conditions. The effects of the physical parameters of the problem such as velocity ratio, Brownian motion, thermophoresis, pressure gradient, and magnetic field, have been investigated in this problem.

2. Governing equations of the problem and similarity analysis

Let us consider a two dimensional unsteady laminar MHD boundary layer flow of viscous incompressible electrically conducting nanofluid over a non-conducting, non-isothermal stretching wedge surface moving with the velocity and the free stream velocity is U(x, t). The positive x-coordinate is measured along the surface of the wedge with the apex as origin, and the positive y-coordinate is measured normal to the x-axis in the outward direction towards the fluid. Taking T_w and T_∞ are the temperature of the wedge wall and free stream of the fluid far away from the wedge respectively, the total angle of the wedge is denoted as $\Omega = \beta r$, where β is the Hartree pressure gradient which are shown in Figure 1. It is assumed that the base fluid and the nanoparticles are in thermal equilibrium and no slip occurs between them. Also, a uniform magnetic field of strength B_0 is introduced to the normal to the direction of the flow. The governing partial differential equations for the boundary-layer flow of nanofluid, can be written as following [16]:

Equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial t} + \nu_f\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho_f}(U - u) \tag{2}$$

Energy equation:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_f\frac{\partial^2 T}{\partial y^2} + \tau\Big\{D_B\Big(\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}\Big) + \frac{\partial T}{T_\infty}\Big(\frac{\partial T}{\partial y}\Big)^2\Big\} \tag{3}$$