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Stability of MHD Wall driven Flow through a Tube

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Abstract

In this work, wall driven incompressible viscous fluid flow through a tube in the presence of magnetic field is considered. The mathematical model of the problem is formulated and solved using series solution method and Hermite-Padé approximation method. Flow becomes unstable due to movement of wall and existence of magnetic field where critical Reynolds number and critical Hartmann number indicate the point of flow instability. This critical point for instability in terms of Reynolds number and Hartmann number are determined and described using bifurcation diagrams. The effect of these two dimensionless numbers on flow velocity and their interaction on flow stability are discussed.

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Keywords: Survey; Motion; IMU

1. Introduction

In this paper, flow is introduced by the movement of wall where this movement promotes the flow instability. Various researchers demonstrate the effect of wall movement on flow structure. Demir and Erturk [5] discussed the stability of non-Newtonian fluids flow through a planar cavity where the behavior of the vortex flow in rectangular cavities ware predicted. The calculations were carried out for different value of the Reynolds number and Weissenberg number. Odejide and Aregbesola [6] examined the incompressible wall driven steady flow of a viscous fluid and heat transfer in a tube. The effect of Reynolds number on wall shear stress and rate of heat transfer were discussed. Makinde et al. [7] observed wall driven steady flow of a viscous fluid and heat transfer through porous tube. The effect of Reynolds number on wall shear stress and the rate of heat transfer across the wall were discussed.

In this problem, magnetic field is applied at the normal to axis of the tube where magnetic field acts as a disturbance on flow and enhances the instability. Some researchers examine the magnetic effect on flow behavior. Makinde and Osalusi [8] investigated the hydro-magnetic steady flow of a viscous fluid in a channel with slip boundaries and solved using perturbation method and Padé approximation technique. The effect of magnetic field on overall flow structure was discussed.

In this paper, we investigate the effect of moving wall of the tube and magnetic field on hydrodynamic stability. The continuity and momentum equations are solved using series solution method. Special type Hermite-Padé approx-

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imation method is used to analyze the series solution. Our concern to detect the critical Reynolds number and critical Hartmann number when the laminar flow breaks down and flow becomes unstable. We also discuss the combine effect of Reynolds number and Hartmann number on flow stability.

2. Mathematical Formulation

Laminar flow of an incompressible viscous fluid through a tube of circular cross section is considered. A polar coordinate system r, z is taken where oz lies along the center of the tube and r is the radial distance. Let u and v be the velocity components in the directions of length and radius of tube respectively, B_0 is the induction of magnetic field that applied along the normal to length of tube. The characterize radius of the tube is a. Characterize axial wall velocity is as shown in Fig. 1.

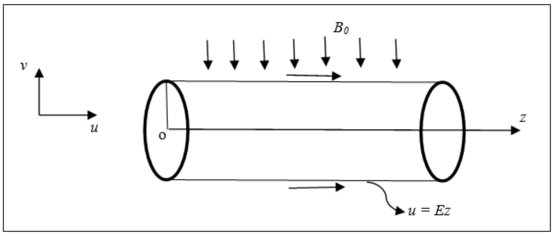

Fig. 1. (a) Direction of ΔP for CW and CCW roll and pitch; (b) Vessel's relative position and heading to earth's latitude and longitude and direction of the resultant deviation in ping position.

The continuity and momentum equations for axisymmetric steady incompressible viscous flow are:

$$\frac{\partial(rv)}{\partial r} + r\frac{\partial u}{\partial z} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \nu(\nabla^2 u) - {B_0}^2\frac{\sigma}{\rho}u \tag{2}$$

$$u\frac{\partial v}{\partial z} + v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \nu(\nabla^2 u - \frac{v}{r^2}) \tag{3}$$

where, $\nabla^2 = \frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}$, P pressure, ρ density, and ν is the kinetic viscosity of the fluid, σ is the electric conductivity
The boundary conditions are:

Along z-axis (center line of the tube)

$$\frac{\partial u}{\partial r} = 0, \ v = 0, \ \text{on } r = 0 \tag{4}$$

At the wall of the tube

$$u = Ez, \ v = 0 \text{ on } r = a \tag{5}$$

Introducing the stream function ψ and vorticity ω as follows:

$$u = \frac{1}{r}\left(\frac{\partial\psi}{\partial r}\right) \qquad v = \frac{1}{r}\left(\frac{\partial\psi}{\partial z}\right) \tag{6}$$

$$\omega = \frac{\partial v}{\partial z} - \frac{\partial u}{\partial r} = -\frac{1}{r}\frac{\partial^2\psi}{\partial z^2} - \frac{1}{r}\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r^2}\frac{\partial\psi}{\partial r} \tag{7}$$

Eliminating pressure P from (2) and (3) by using (6), (7) we have

$$\frac{1}{r}\left(\frac{\partial\psi}{\partial r}\frac{\partial\omega}{\partial z} - \frac{\partial\psi}{\partial z}\frac{\partial\omega}{\partial r}\right) + \frac{\omega}{r^2}\frac{\partial\psi}{\partial z} - \frac{{B_0}^2\sigma}{\rho}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\psi}{\partial r}\right) = \nu\left(\nabla^2\omega - \frac{\omega}{r^2}\right) \tag{8}$$