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ScienceDirect

Procedia Engineering 190 (2017) 178 – 185

Procedia Engineering

www.elsevier.com/locate/procedia

Structural and Physical Aspects of Construction Engineering

Nonlocal and Gradient Theories of Piezoelectric Nanoplates

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Abstract

Classical continuum models are unable to describe the size effect in nano/micro structures, even though this effect is observed experimentally. Therefore, modified continuum models are frequently applied for the investigation of nanomechanics due to their computational efficiency and capability of providing accurate results which are comparable to the atomistic models. In this paper, the Mindlin plate is extended to the piezoelectric nanoplate with nonlocal and gradient theories for size effect. The governing equations for bending moments, normal and shear stresses are derived via the Hamilton`s principle. Differences between the two theories are described.

Peer-review under responsibility of the organizing committee of SPACE 2016

Keywords: Nonlocal; gradient elasticity; Mindlin plate; Hamilton`s principle; size effect

1. Introduction

Over the past two decades, nanostructures have attracted much attention among the scientific research community. Due to their superior features, the application of nano/micro structures was expanded into many areas such as nano-electromechanical devices, space and bio-engineering, actuators, and nanocomposite. To make the design of nanostructures in a real manner, it is important to study the mechanics of systems at nanometer scale. Experimental techniques as well as discrete atomistic methods such as molecular dynamics (MD) simulations are either extremely difficult or highly expensive. Size effect phenomenon can be observed if the component dimension is comparable to the material length scale [1-4]. Modified continuum models are frequently applied for the investigation of nanomechanics due to their computational efficiency and the capability of providing accurate results which are comparable to atomistic models. The couple-stress theory [5,6] is one of the higher-order theories, which contains two length-scale parameters. A modified couple-stress theory [7] contains only one material length-scale parameter.

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Peer-review under responsibility of the organizing committee of SPACE 2016
doi:10.1016/j.proeng.2017.05.324

Li and Pan [8] applied this theory for size-dependent functionally graded piezoelectric microplate. The same approach was applied by Jomehzadeh et al. [9] to study the vibration of microplates described by the Kirchhoff plate model. Ma et al. [10] developed a non-classical Mindlin plate model based on a modified couple-stress theory. A theory for nano-sized elastic dielectrics with the electric field-gradient and electrostatic force effect based on a variational principle was developed by Hu and Shen [11]. Size effect due to both the electric field gradient and surface effects in dielectrics was considered there. These authors also developed a more sophisticated theory, where the flexoelectricity and electrostatic force effects were included [12]. Liang et al. [4] presented the theoretical investigation of the size-dependent Bernoulli-Euler dielectric nanobeam described by the strain gradient elasticity.

The other way to consider the size effect for micro/nano structures is the nonlocal elasticity theory proposed by Eringen [13]. In this nonlocal theory, the stress at a reference point is a functional of strains at more points of the body. The nonlocal elasticity theory explains satisfactorily some phenomena such as the high frequency vibration and wave dispersion. A review of the literature indicates that the nonlocal theory is frequently applied for various nanostructures such as the carbon nanotubes, graphene sheets and piezoelectric nanoplates [14]. The free vibration of Mindlin rectangular nanoplates was analyzed by Hosseini-Hashemi et al. [15] by introducing some auxiliary and potential functions. The free vibration of size-dependent magneto-electro-elastic nanobeams was studied by Ke and Wang [16]. The thermo-electro-mechanical free vibration of piezoelectric nanobeams was investigated by Ke and Wang [17]. The Galerkin finite element formulation for nonlocal elastic Euler–Bernoulli beam and Kirchoff plate was presented by Phadikar and Pradhan [18]. Liu et al. [19] analyzed free vibration of piezoelectric nanoplates described by Kirchhoff theory and nonlocal elasticity. An efficient computational approach based on a refined plate theory including the thickness stretching, namely quasi-3D theory, in conjunction with isogeometric formulation was proposed by Nguyen et al. [20] for the size-dependent bending, free vibration and buckling analysis of functionally graded nanoplate structures. Sobhy [21] investigated the bending response, free vibration, mechanical buckling and thermal buckling of functionally graded material (FGM) nanoplate which is located in an elastic medium.

One can see that both nonlocal and gradient theories have been applied to the analyses of piezoelectric nanoplates. The goal of the present paper is to compare both theories and to show their differences. The Mindlin plate theory is described by the nonlocal and gradient elasticity for piezoelectric nanoplates. The governing equations for the bending moment, normal and shear stresses are derived from the Hamilton`s principle. The electric field along the plate thickness can be expressed through the gradients of the plate rotations. The governing equations for harmonic oscillation are also obtained from the general time dependent formulation.

2. Nonlocal theory for Mindlin plate

We consider a plate of thickness *h* with homogeneous piezoelectric material properties with its mean surface occupying domain Ω in the (x_1, x_2) -plane. The axis $x_3 \equiv z$ is perpendicular to the mid-plane (Fig.1) with the origin at the mid plane. The Cartesian coordinate system is introduced such that the bottom and top surfaces of the plate is placed in the planes z = -*h*/2 and z = *h*/2, respectively. The linear strains are given by [22]:

$$\varepsilon_{11}(\mathbf{x}, x_3, \tau) = u_{10,1} + z w_{1,1}(\mathbf{x}, \tau), \qquad \varepsilon_{22}(\mathbf{x}, x_3, \tau) = u_{20,2} + z w_{2,2}(\mathbf{x}, \tau),$$

$$\varepsilon_{12}(\mathbf{x}, x_3, \tau) = \frac{1}{2}(u_{10,2} + u_{20,1}) + \frac{1}{2} z \left[w_{1,2}(\mathbf{x}, \tau) + w_{2,1}(\mathbf{x}, \tau) \right], \qquad \varepsilon_{13}(\mathbf{x}, \tau) = \left[w_1(\mathbf{x}, \tau) + w_{3,1}(\mathbf{x}, \tau) \right] / 2,$$

$$\varepsilon_{23}(\mathbf{x}, \tau) = \left[w_2(\mathbf{x}, \tau) + w_{3,2}(\mathbf{x}, \tau) \right] / 2, \tag{1}$$

where the in-plane displacements in x_1 - and x_2 -directions are denoted by u_{10} and u_{20} . Rotations around x_2 - and x_1 -axes are denoted by w_1 and w_2 , and w_3 is the out-of-plane deflection.

In nonlocal elastic theory the stress at a reference point x depends on the strain at other points x` in the vicinity of x [23]. Accordingly, the basic constitutive equations for a piezoelectric solid are given by

$$\sigma_{ij}(\mathbf{x}, \tau) = \int_\Omega \alpha(|\mathbf{x}' - \mathbf{x}|, \zeta) \left[c_{ijkl} \varepsilon_{kl}(\mathbf{x}', \tau) - e_{kij} E_k(\mathbf{x}', \tau) \right] d\mathbf{x}', \tag{2}$$

$$D_j(\mathbf{x}, \tau) = \int_\Omega \alpha(|\mathbf{x}' - \mathbf{x}|, \zeta) \left[e_{jkl} \varepsilon_{kl}(\mathbf{x}', \tau) + h_{jk} E_k(\mathbf{x}', \tau) \right] d\mathbf{x}', \tag{3}$$

where $\{\varepsilon_{ij}, E_i\}$ is the set of the secondary field quantities (strain, electric field) which are expressed in terms of the