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Statics and Dynamics of Snap-through Effect

Ľuboš Šnirc[a,*], Ján Ravinger[b]

[a,b] *Department of Structural Mechanics, Faculty of Civil Engineering, Slovak University of Technology, 810 05 Bratislava, Radlinského 11, Slovakia*

Abstract

Snap-through effect i.e. sudden change of buckling surface is unpleasant phenomenon in behavior of structures especially in thin-walled structures. Geometric non-linear theory (The Total Lagrange Description) must be used. In geometric non-linear solution we have more results and snap-through effect can be explained as a jump between branches of solutions. It can be smooth but mostly this effect is sudden and accompanied with big noise. Examples of imperfect columns, von-Mises truss, shallow arch and steel thin-walled panels are presented.

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Keywords: Stability; buckling; post-buckling; dynamic stability; vibration; geometric non-linear theory; initial imperfections; thin-walled structures

1. Introduction

We used to say that Leonard Euler established the problem of stability 250 years ago. Presented paper is oriented into combination of stability and dynamics. Problem of the combination of linear stability (ideal structure) and vibration was solved a long time ago. Differences, between theoretical results and the reality, forced researchers to search for more accurate models. The slender web as a main part of thin-walled structure has significant post-buckling reserves and for a description of them it is necessary to accept a geometric non-linear theory. Burgeen (1951) formulated the problem of vibration of an imperfect column. The problem of vibration of slender web as a non-linear system was formulated by Bolotin (1956). The vibration of rectangular slender web taking into account geometrical initial imperfections has been investigated by many researchers (Wedel-Heinen, J. 1991, Hui, D. 1984, Ilanko, S. and Dickinson, S. M. 1991, Yamaki, V. at al. 1983). Ravinger (2012) arranged the general theory for the

* Corresponding author. Tel.: 0042159274257.
E-mail address: lubos.snirc@stuba.sk

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dynamic post-buckling behavior. Sudden change in buckling surface (the snap-through effect) is a special problem occurring in stability of structures.

2. Theory

Von Kármán theory has been used for the description of post-buckling behavior of a thin-walled panel with geometrical imperfections and residual stresses. By including inertia forces the problem is extended into dynamics. A direct formulation of Hamilton's principle for the dynamic post-buckling behavior of a slender web leads to the system of conditionals equation describing this non-linear dynamic process. In the case of static problem, linearized assumptions are accepted for the evaluation of the elastic critical load. Analogously, in the case of dynamic problem, Hamilton's principle is arranged in incremental form for the evaluation of free vibration. The system of conditional equations in an incremental formulation for post-buckling behavior of the slender web or the thin-walled structure can be written as

$$\boldsymbol{K}_{MP}\Delta\ddot{\alpha}_P + \boldsymbol{K}_{INCP}\Delta\alpha_D + \boldsymbol{K}^T_{INCWP}\Delta\alpha_W + \boldsymbol{F}_{INTP} - \boldsymbol{F}_{EXTP} - \Delta\boldsymbol{F}_{EXTP} = \boldsymbol{0} \tag{1a}$$

$$\boldsymbol{K}_{INCW}\Delta\alpha_W + \boldsymbol{K}_{INCWP}\Delta\alpha_P + \boldsymbol{F}_{INTW} - \boldsymbol{F}_{EXTW} - \Delta\boldsymbol{F}_{EXTW} = \boldsymbol{0} \tag{1b}$$

where indexes mean $_M$ - mass, $_{INC}$ - incremental, $_{INT}$ – internal, $_{EXT}$ – external, $_P$ – plate, $_W$ - web.
The incremental stiffness matrix of the web $\boldsymbol{K}_{INCW}$ is linear and, consequently, we can proceed to the elimination of Eqs.(1b) and thus obtain the system of equations

$$\boldsymbol{K}_{MP}\Delta\ddot{\alpha}_P + (\boldsymbol{K}_{INCP} - \boldsymbol{K}^T_{INCWP}\boldsymbol{K}^{-1}_{INCW}\boldsymbol{K}_{INCWP})\Delta\alpha_P + \boldsymbol{F}_{INTP} - \boldsymbol{F}_{EXTP} -$$

$$\boldsymbol{K}^T_{INCWP}\boldsymbol{K}^{-1}_{INCW}\boldsymbol{F}_{INTW} + \boldsymbol{K}^T_{INCWP}\boldsymbol{K}^{-1}_{INCWP}\boldsymbol{F}_{EXTW} - \Delta\boldsymbol{F}_{EXTP} + \boldsymbol{K}^T_{INCWP}\boldsymbol{K}^{-1}_{INCW}\Delta\boldsymbol{F}_{EXTW} = \boldsymbol{0} \tag{2}$$

Taking out the inertia forces we get conditional equations describing the static post-buckling behavior of a slender web or large displacements of plates. The Newton-Raphson iteration (N-R iteration) can be used for the solution of these equations. From a number of rules whose are valid in numerical models of static non-linear problems we use one which can help us in establishing the free vibration problem.

$$\boldsymbol{K}_{INC} \equiv \boldsymbol{J} \quad \Rightarrow \quad \boldsymbol{K}_{INC} = \boldsymbol{K}_{INCP} - \boldsymbol{K}^T_{INCWP}\boldsymbol{K}^{-1}_{INCw}\boldsymbol{K}_{INCWP} \tag{3}$$

This means that the incremental stiffness matrix is equal to the Jacobian of N-R iteration of the system of non-linear algebraic equations. We must note that this rule valid only if the parameter of load increment is the pivot term in N-R iteration. During the snap-through effect the incremental stiffness matrix is singular and we are not able to arrange N-R iteration. To overcome this level of the load, the displacement parameters must be used as pivot term. The problem of the ***free vibration*** including the effects of initial imperfections (initial displacement and residual stresses) can be obtained in the following way. If we suppose the system Eqs.(2) in equilibrium, then

$$\boldsymbol{F}_{INTP} - \boldsymbol{K}^T_{INCWP}\boldsymbol{K}^{-1}_{INCWP}\boldsymbol{F}_{EXTW} - \Delta\boldsymbol{F}_{EXTP} + \boldsymbol{K}^T_{INCWP}\boldsymbol{K}^{-1}_{INCW}\Delta\boldsymbol{F}_{EXTW} = \boldsymbol{0} \tag{4}$$

We suppose a zero increments in the external load

$$\Delta\boldsymbol{F}_{EXTP} = \Delta\boldsymbol{F}_{EXTW} = \boldsymbol{0} \tag{5}$$

The increments of the plate displacements are assumed as

$$\Delta\alpha_P = \Delta\overline{\alpha}_P \sin(\omega t) \tag{6}$$

where ω is the natural frequency of the free vibration.
Inserting this into Eqn.(2) we have a problem of eigenvalues and eigenvectors

$$\left|\boldsymbol{K}_{INC} - \omega^2\boldsymbol{K}_{MP}\right|_{det} = 0 \tag{7}$$