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Wavelet Approach to Damage Detection of Mechanical Systems and Structures

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Abstract

The damage detection methods of mechanical and civil structures have been drawing much interest from various fields. Many techniques to detect damage are based on the examination of the system response signals. Crack-like damages may contribute to the response signal the edges (also called slopes), i.e. the localized sharp transitions of signal values. A powerful tool to characterize such local feature signals is the continuous wavelet transform (CWT). Despite its name, the CWT can be calculated on discrete data and requires enormous computational cost. However, in many cases, the crack-like changes in the signal can be detected and localized using the discrete dyadic wavelet transform (DDWT) that has a fast transform algorithm. This paper presents application of the DDWT to detect and localize the response signal features (slopes) due to cracks in mechanical and civil structures. The wavelets used to calculate the DDWT are cubic box splines. The numerical results for a response signal simulating cracks in structures are presented in this article.

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Keywords: damage detection; wavelet; discrete dyadic wavelet transform; cubic box spline wavelets

1. Introduction

Wavelet based approaches to damage detection were presented in many papers [1–5]. The literature review of the research that has been conducted on damage detection by wavelet analysis is given in references [1,3]. The authors of the paper [1] claime that "Wavelet-based damage identification approaches are primarily associated with detecting singularities in the response signal either in space or in time or any of their derivative".

This paper presents just an approach to damage detection and identification based on Mallat's wavelet theory of signal singularities [6]. Our method requires the knowledge of the response signal of damaged system and bases on the assumption that the response signal contains the slopes (localized sharp transitions of signal values) caused by the damage. The measurement methods of such a signal are out of the scope of this paper.

Slopes of a signal can be localized by examination of the signal derivative. The modulus of signal derivative is large in the neighbourhood of the points where the signal varies rapidly in value. However, numerical differentiation

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of a discrete-time signal is an ill-conditioned problem. One can reduce numerical erros by the preliminary smoothing the signal; derivative of the smoothed signal is equal to a convolution of the signal with the derivative of smoothing function [6]

$$\frac{d}{dt}\left[f \star \overline{\theta}(t)\right] = f \star \overline{\theta}'(t), \tag{1}$$

where $\theta(t)$ - smoothing function, $\overline{\theta}(t) = \theta(-t)$ and $\overline{\theta}'(t) = \frac{d\bar{\theta}(t)}{dt}$.

The signal can be smoothed by a scaled smoothing function. The scaled function $f_s(t)$ is defined as

$$f_s(t) = \frac{1}{\sqrt{s}}\, f\left(\frac{t}{s}\right). \tag{2}$$

Derivative of the signal smoothed by the scaled smoothing function is equal to

$$\frac{d}{dt}\left[f \star \overline{\theta}_s(t)\right] = \frac{1}{s}\, f \star \overline{\theta}_s'(t). \tag{3}$$

The last equation shows that the derivative of a signal smoothed by the scaled smoothing function is equal to a convolution of the signal with a derivative of the scaled smoothing function (with the accuracy to coefficient $1/s$).

Smoothing introduces to a signal inflection points that correspond to the slopes of the signal. Localizing the inflection points we localize the slopes of the signal. The first derivative of the smoothed signal has local extremes in inflection points and the second one has zero-crossings. Thus, calculating the points where the first derivative of the smoothed signal has local exteremes we localize the slopes of the signal. The second way to localize the slopes is calculating the zero-crossings of second derivative of the smoothed signal.

Replacing in (1) $\overline{\theta}'(t)$ by $\psi(t)$ we recive

$$\frac{d}{dt}\left[f \star \overline{\theta}(t)\right] = f \star \overline{\psi}(t), \tag{4}$$

where $\psi(t) = -\theta'(t)$ can be interpreted as a wavelet and the right side of that equation is a symbolic form of the wavelet transform of signal. In concsequence, in the case of wavelet which is the first derivative of the smoothing function, slopes correspond to the modulus maxima of the wavelet transform; in the case of wavelet which is the second derivative of the smoothing function, slopes correspond to the zero-crossings of the wavelet transform. The mathematical foundations for signal slope detection based on the wavelet transform were presented by Stéphane Mallat in the papers [7–9]. The next section gives a brief summary of the basis of that approach to slope detection.

2. Wavelet theory

A wavelet signal representation is based on two functions: a scaling function $\phi(t)$ and a wavelet $\psi(t)$ and two digital filters: a lowpass filter h and a highpass filter g. In terms of signal processing, the scaling function is an impulse response of a lowpass filter and the wavelet is an impulse response of a bandpass filter. Both wavelet and scaling function are characterized by fast decay, e.g. exponential, or compact support. The relations between the scaling function, the wavelet and the filters are expressed by the following equations:

$$\frac{1}{\sqrt{2}}\,\phi(t/2) = \sum_n h[n]\,\phi(t-n), \tag{5}$$

$$\frac{1}{\sqrt{2}}\,\psi(t/2) = \sum_n g[n]\,\phi(t-n). \tag{6}$$

The equation (5) is called the dilation equation. A characteristic feature of the scaling function is a non-zero value of the following integral

$$\int_{-\infty}^{+\infty} \phi(t)\,dt \neq 0. \tag{7}$$