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Magic Formula Tyre Model Application for a Tyre-Ice Interaction

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Abstract

Magic Formula (MF) tyre model is widely used for the analysis of tyre behavior in different driving situations. The model is mostly used to fit the simulation and experimental data slip curves. As there are many modifications of the MF, this paper presents a literature analysis of MF application and fitting techniques. A short review of other empirical tyre models was also made. At the end, a simple least squares minimisation technique was used to fit the experimental data of longitudinal tyre performance on ice. Measurements were performed with the inner drum test rig in the laboratory. In general, the fitting showed a very good accuracy.

Keywords: Magic Formula, tyre, slip ratio, ice, least squares minimisation

1. Introduction

Tyre models are a prerequisite for any vehicle dynamics simulation and range from the simplest mathematical models that consider only the cornering stiffness to a complex set of formulae. Among all the steady-state tyre models that are in use today, the Magic Formula (MF) tyre model is unique and most popular. Though the MF tyre model is widely used, obtaining the model coefficients from either the experimental or the simulation data is not straightforward due to its nonlinear nature and the presence of a large number of coefficients. A common procedure used for this extraction is the least-squares minimisation that requires considerable experience for initial guesses [1].

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The parameterization of MF model sometimes aren' simply adaptable to other track surfaces, like winter tracks. This paper presents a review of other empirical models, MF tyre model application and techniques of coefficient determination. Then a least squares minimisation technique is used for fitting the experimental tyre-ice interaction data obtained from the indoor measurements.

2. Review of empirical models

Burkhardt [2] developed the model, where friction coefficient μ is expressed as a function of the wheel slip ratio s, and the vehicle velocity v.

$$\mu(s,v)=\left[C_1\left(1-e^{-C_2 s}\right)-C_3 s\right]e^{-C_4 s v}, \tag{1}$$

where C_1 is the maximum value of friction curve; C_2 is the friction curve shape; C_3 is the friction curve difference between the maximum value and the value at $s = 1$; C_4 is wetness characteristic value. By changing values of parameters $C_1 - C_4$, many different tire-road friction conditions can be modelled. The parameters for different road surfaces are listed in Table 1.

Table 1. Burckhardt tyre model parameters [2].

Surface conditions	C_1	C_2	C_3	C_4
Dry asphalt	1.029	17.16	0.523	0.03
Dry concrete	1.197	25.168	0.5373	0.03
Snow	0.1946	94.129	0.0646	0.03
Ice	0.05	306.39	0	0.03

Kiencke and Daiss [3] expanded and approximated previous Burkhardt model and suggested a simpler model.

$$F(s)=k_s\frac{s}{c_1 s^2+c_2 s+1}, \tag{2}$$

where k_s is the slope of the $F(s)$ versus s curve at $s = 0$, and c_1 and c_2 properly chosen parameters.

All the previous friction models are highly nonlinear in the unknown parameters, and thus they are not well-adapted to be used for on-line identification. For this reason, simplified models like

$$F(s)=c_1\sqrt{s}-c_2 s, \tag{3}$$

have been proposed [4].

It is also well known that the 'constant' $c'_i s$ in the above models, are not really invariant, but they may strongly depend on the tire characteristics (e.g., compound, tread type, tread depth, inflation pressure, temperature), on the road conditions (e.g., type of surface, texture, drainage, capacity, temperature, lubricant, etc.), and on the vehicle operational conditions (velocity, load) [4].

Jazar [5] suggested empirical equation to show the effects of pressure p and load F_z on the rolling friction coefficient μ_r.

$$\mu_r=\frac{K}{1000}\left(5.1+\frac{5.5\cdot 10^5+90F_z\cdot}{p}+\frac{1100+0.0388F_z}{p}v_x^2\right). \tag{4}$$

The parameter K is equal to 0.8 for radial tires, and is equal to 1.0 for non-radial tires.