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Rod resonant oscillations considering material relaxation properties

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Abstract

The mathematical model of oscillations of an elastic rod under external harmonic load considering the material relaxation properties and the resistance of the rod towards the process of its deformation has been developed. The model differential equation is derived considering the dependence on resistance time and deformations according to the formula for Hooke's law brought to Maxwell's and Kelvin-Voight's elaborated models. The results of numerical analysis of the model show that the coincidence of the rod oscillation frequency with the external load oscillation frequency results in resonance followed by the unlimited growth of the amplitude oscillation (providing that there is no environment resistance). If the environment resistance and material relaxation behavior are considered, and the rod oscillation frequency coincides with the oscillation frequency of the internal load (resonant oscillations), the types of the oscillation process may be as follows. The oscillations damp in time (at low values of the relaxation factors and high values of the environment resistance factors). The oscillations become stabilized reaching some permanent state of the amplitude (undamped oscillations). At some high values of the relaxation factors and low values of the resistance factors the resonant frequencies are followed by the bifurcation resonance effects in the undamped oscillation processes, and the non-resonant frequencies are followed by beating in the damped oscillation processes. At some higher values of the resistance factor of the rod material, regardless of the resonant frequencies, the restore of an unbalanced rod takes place practically without the oscillation process – only the internal load oscillations remain constant.

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Keywords: rod oscillations, hyperbolic equation, numerical solution, stress-strain relaxa-tion, relaxation factor, resistance factor, resonant oscillations.

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1. Introduction

Elastic deformation of a solid body caused by some disturbance propagates at the speed which is dependent on the environment properties. However, the wave process of the environment oscillation is not followed by a substance movement. The equations presenting these processes are of a hyperbolic type. In relation to technical devices there is a common case when the free oscillation process initiated by some initial displacement is followed by a load applied to the loose end of the rod which functions according to a specific law. The matter of a particular interest is determination of an increase in resonance of the oscillation amplitude, when the free oscillation frequency coincides with the oscillation frequency of the load applied to the rod [1, 2].

Derivation of a differential equation of the rod oscillations is based on Hooke's law

$$\sigma = E\partial U / \partial x \ , \tag{1}$$

and Newton's second law presented as a motion equation

$$\partial\sigma / \partial x = \rho\partial^2 U / \partial t^2 . \tag{2}$$

where σ is normal stress, N/m^2 ; U is motion, m ; x is a coordinate, m ; t is time, sec ; ρ is density, kg/m^3 ; E is the elastic modulus (Young's modulus), Pa ; $\varepsilon = \partial U / \partial x$ is deformation, m .

If we substitute the formula (1) into the formula (2), we will find [2, 3]

$$\frac{\partial^2 U(x,t)}{\partial t^2} = e^2 \frac{\partial^2 U(x,t)}{\partial x^2} , \tag{3}$$

where $e = \sqrt{E/\rho}$ is a speed of the longitudinal disturbance propagation, m/sec .

The equation (3) is a wave hyperbolic equation presenting undamped oscillations of elastic bodies. The absence of damping is due to the absence of the summand in the equation which considers the internal resistance of the environment affected by the mechanic load causing elastic motions. To consider the environment resistance we shall accept that F_c which stands for the resistance force is proportional to displacement velocity in time

$$F_c = -r\partial U / \partial t , \tag{4}$$

where r is a resistance coefficient, kg/sec .

The minus in the formula (4) means that the resistance force has the direction which is opposite to the displacement velocity.

If we substitute the formula (4) in the equation of Newton's second law, we will find

$$F = ma = m\frac{d\upsilon}{dt} = m\frac{d^2 U}{dt^2} = \rho S\Delta x \frac{d^2 U}{dt^2} , \tag{5}$$

considering that the resistance force F_c relates to the volume forces, we will find

$$\rho \frac{d^2 U}{dt^2} = \frac{d\sigma}{dx} - \frac{r}{V}\frac{dU}{dt} , \tag{6}$$

where F is a force affecting the body, $kg \cdot m/sec^2$; m is a mass of the body, kg ; $a = d\upsilon/dt$ is acceleration, m/sec^2 ; $\upsilon = dU/dt$ is velocity, m/sec ; S is a cross section area of the body, m^2 ; Δx is the length of a surface element, m ; V is volume, m^3 .