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Estimation of Origin-Destination Matrices Based on Markov Chains

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Abstract

The method of estimating origin-destination matrices of correspondence using observational data on traffic flows based on the Markov chain theory is considered in this paper. The method is based on the transportation network, which is associated with the graph of the corresponding Markov chain and on the canonical form of the graph proposed. The classification of observation models on flows in a transportation network is provided. The properties of the method proposed are investigated on several simple networks. The recommendations for the practical application of the method proposed in real transportation networks are given.

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Keywords: transport modelling, origin-destination matrices, Markov chains

1. Introduction

Mathematical equilibrium models of traffic flows are widely used to support decision-making process in the management of transport systems of large cities, agglomerations. The development of these models can be divided into two stages. The first stage is to estimate origin-destination (OD) matrices based on the initial data. At the second stage the origin-destination matrices obtained are distributed to the transportation network. Elements of an OD matrix area total number of users moving from one point of the transportation network to another. The problem

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of obtaining origin-destination flows is that they are not explicitly observable, and we have to find them indirectly. Different methods that depend on the initial data are commonly used to estimate OD flows. And all methods can be divided into 2 classes. The first class includes gravity and entropy models described by Vasil'eva *et al.* (1981) and by Wilson (1967). The gravity model is used by analogy with the famous law of gravitation; the entropy model is based on a maximum entropy method. A significant disadvantage of these methods is that they are described by a common function whose parameters are determined based on the mobility of the population, i.e. the indirect information about movements of citizens.

The second class of methods involves the estimation of OD flows using the observations on the traffic flow. Vardi (1996) defined this class of methods as "network tomography". A variety of such methods has been already developed. Tebaldi and West (1998), Li (2005) considered the Bayesian approach to the problem of OD matrix estimation in their papers. Hazelton (2001) conducted a comprehensive study of the problem and identified its fundamental issues. All the methods above assume that a so-called assignment matrix exists, i.e. the matrix that defines the connection between origin-destination flows and their paths, and between paths and links in which traffic flows are observed. Another approach to the problem of network tomography is based on a Markov representation of transport flows (not a Monte Carlo Markov chain (MCMC) method). In the papers of Crisostomi *et al.* (2010) and Morimura *et al.* (2013) approaches to the description of origin-destination trips using Markov chains were presented. Li (2009) considered a method to estimate OD matrices of the public transport using Markov approach. Khabarov *et al.* (2012) used Markovian approach to assess the OD matrix by the measurements on borders of transport zones. In this paper, we consider the approach to the estimation of OD matrices using observations on traffic flows on network nodes based on a Markov representation of transport OD flows.

In the section 2.1, the model of a transportation network is considered, and the classification of observation models on flows in a transportation network for the problems of estimation OD flows is provided. In the section 2.2, the possibility of presenting a transportation network as a Markov chain is considered, and a method for obtaining OD flows based on their Markov properties is provided. In the section 2.3, the properties of the method proposed are studied. Section 2.4 discusses practical aspects of the application of the method proposed. Section 3 is the conclusion.

2. Estimation of OD matrices

2.1. Observation models

Let us present a transportation network as a directed weighted graph **G** (**V**, **E**), where **V** is the set of vertices, and **E** is the set of edges. The transportation network **G** (**V**, **E**) is associated with a physical transportation network. The vertices are the transportation network nodes (junctions, interchanges, origins and destinations of trips), and the edges are network links, i.e. some connections between nodes. Some microscopic objects, such as cars, move through the transportation network and form a traffic flow. There are some physical and technological limitations in observing the transportation network. These restrictions give rise to 5 basic observation models in the graph **G**:

1. *The observation model at the network node*. A total number of micro-objects located in the node $v \in \mathbf{V}$ are observed during the time interval t: $n_v(t)$.
2. *The observation model on the network link*. A total number of micro-objects on the link $e \in \mathbf{E}$ of the transportation network **G** is observed during the time interval t: $n_e(t)$.
3. *The observation model at the network turns*. For such an observation model it is necessary to introduce the concept of the dual graph **L** (**G**) described by Harary (1969). Vertices of the dual graph **L** (**G**) are associated with the edges of the graph **G**. Consequently, edges of the dual graph **L** (**G**) connect vertices that correspond to adjacent edges of the graph **G** (see. Fig. 1). A total number of micro-objects on the link $e_l \in \mathbf{L}(\mathbf{E})$ of the dual **L** (**G**) is observed during the time interval t: $n_{el}(t)$. It can be interpreted as the flow intensity of some turn.
4. *The observation model at the network route*. The route is the k-th degree turn. The k-th degree turn, where $k > 1$, means the number of micro-objects passing through a chain of k adjacent links of **G**. A total number of micro-objects on the link $q \in \mathbf{L}^k(\mathbf{G}) = \mathbf{L}(\mathbf{L}^{k-1}(\mathbf{G}))$ (see. Fig. 1) is observed during the time interval t: $n_q(t)$. It may be interpreted as the intensity of the k-th degree turns.