Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 186 (2017) 252 – 260

Procedia Engineering

www.elsevier.com/locate/procedia

XVIII International Conference on Water Distribution Systems Analysis, WDSA2016

Sensitivity Analysis of Topological Subgraphs within Water Distribution Systems

Jochen Deuerlein[a]*, Olivier Piller[b], Sylvan Elhay[c], Angus Simpson[d]

[a]3S Consult GmbH, Albtalstrasse 13, 76137 Karlsruhe, Germany
[b]Irstea, UR ETBX, Water Department, Bordeaux regional centre, Cestas F-33612, France
[c]School of Computer Science, University of Adelaide, SA 5005,
[d]School of Civil, Environmental and Mining Engineering, University of Adelaide, Adelaide SA 5005, Australia

Abstract

Sensitivity analysis of the actual hydraulic state of water distribution systems is a valuable tool with number of applications in hydraulic systems analysis. Sensitivity matrices include the information of the response of the hydraulic state variables (flows, pressures) to changes in model parameters (e.g. demands, roughness, control parameters) for a specific hydraulic state of the system. For calculation, there exists in addition to finite difference approximations also exact solutions that include the inversion of the system matrix (the Schur Complement of the Jacobian of the hydraulic network equations). In combination with hydraulic network simulations, the factorization (for example Cholesky matrix decomposition) of the matrix that has already been done by the hydraulic solver in the computation process can be used for the efficient calculation of the inverse matrix. However, for large realworld networks the sensitivity calculation is a time and memory consuming process because the inverse of the system matrix of a connected network has no zero elements. In this paper a new method is presented that allows for the exact calculation of sensitivities of a particular subgraph of interest, the topological minor or supergraph within a water distribution system network graph. Supernodes are the most important nodes in terms of connectivity redundancy within the network graph. Superlinks replace all pipes (links) in series between two supernodes. It will be shown that the sensitivities that are calculated for the subgraph deliver exactly the same results as the inversion of the entire system matrix reduced to supernodes. This paper focuses on the derivation of the equations for the reduced system matrix inversion of the topological subgraph. In addition, the paper includes the proof of equivalence of the matrix inverses for the topological minor subgraph. This inverse represents the fundamental sensitivities of nodal heads and pipe flows with respect to nodal demands in demand driven analysis. The results presented can be extended to other sensitivities, since the matrix inverse in question is included in all other derived parameter sensitivities.

* Corresponding author. Tel.: +49 721 33503360; fax: +49 721 335033130.
E-mail address: deuerlein@3sconsult.de

Peer-review under responsibility of the organizing committee of the XVIII International Conference on Water Distribution Systems
doi:10.1016/j.proeng.2017.03.239

Peer-review under responsibility of the organizing committee of the XVIII International Conference on Water Distribution Systems

Keywords: Water distribution systems, sensitivity analysis, topological minor, matrix inversion;

1. Introduction

Sensitivity analysis plays an important role in hydraulic system engineering. Sensitivities are first order estimates of the change of the state variables (flows, heads) with respect to different kind of parameter changes. Consequently, sensitivity analysis can be used for example for hydraulic model calibration [1]. The calculation of the sensitivities is normally a computationally costly procedure since it involves the inversion of the Schur Complement $\mathbf{J} = \mathbf{A}^T\mathbf{F}^{-1}\mathbf{A}$ of the Jacobian matrix of the system equations of the Global Gradient Algorithm [2]. In this paper a new method is proposed that enables the calculation of the analytically exact sensitivities for a smaller subset of nodes. It will be proven that these sensitivities are equivalent with those that results from the inversion of the full system matrix. In terms of time saving, this is a very important result.

In a previous paper the Graph Matrix Partitioning Algorithm (GMPA) was introduced [3]. It was shown that the hydraulic steady-state calculation of large complex pipe networks can be split into a local and a global solution. Whereas the global solution includes solving a nonlinear system of equations the local solution consists of simple linear calculations. The basic idea of this method was the observation that supply networks commonly include a number of tree-like subgraphs (e.g. the large number of subsystems representing the secondary distribution and house connection pipes) that can be treated separately in a more efficient manner. The nodes of these subsystems often carry important information about withdrawals and cannot be removed without losing accuracy. With the GMPA exact accuracy (where the solution is completely identical to the full solution of the hydraulic equations) is maintained while reducing the size of the system to be solved by magnitude.

In this paper the basic idea of the GMPA is used for the derivation of the sensitivity matrix of the global topological minor subgraph. The development is based on the two assumptions that, first, the graph theoretical forest has been removed from the system and, second, that only Demand Driven Analysis (DDA) is considered.

In [3] the topological minor subgraph of a network graph that consists of supernodes and superlinks was introduced. Whereas the set of supernodes is a real subset of the original set of nodes, the superlinks replace the series of original links between the two supernodes. Fig. 1 shows the network graph of an example system (left) and its topological minor (right). The supernodes are the reference node R (by definition) and nodes a and b. The identification of the supernodes is simple for the network core: all nodes with nodal degree > 2. The superlinks consist of the paths between the supernodes. For example, in Fig. 1, superlink s2 replaces the original links 2, 3 and 4. If for each superlink one arbitrary link has to be chosen as so called internal tree chord, the links can be subdivided into the internal tree chord and an arbitrary number of internal tree links.