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Advances in the Theory of Surface Growth with Applications to Additive Manufacturing Technologies

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Abstract

A vast majority of objects or solids that surround us arise from some surface growth processes. As an example, one can present many technologies in industry, including well-known technologies of crystal growth, laser deposition, solidification of melts, electrolytic formation, pyrolytic deposition, polymerization, concreting, and modern digital additive manufacturing technologies. Similar processes determine the specific features of natural phenomena such as the growth of biological tissues, glaciers, blocks of sedimentary and volcanic rocks, space objects, etc.

Recent research has shown that solids formed by growth processes differ in their properties essentially from solids in the traditional view. Moreover, the classical approaches of solid mechanics to modeling the growing solids behavior fail. They should be replaced by new ideas and methods of modern mechanics, mathematics, physics, and engineering sciences. An approach proposed deals with the construction of adequate model of surface growth processes of solids and its application to mechanical problems of additive manufacturing technologies.

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1. Preliminary Remarks

Deformation processes in a solid whose composition, mass, or volume varies in a piecewise continuous manner due to the influx of new material is of great interest for engineers, researchers and technologists in numerous areas. The solid mechanics problems arising in the field of modeling of such processes are completely new and form a separate field of research known as mechanics of growing solids. Its importance is determined by the fact that almost all solid objects in nature and technology (buildings, structures, structural components, machine parts, trees, bones, soft tissues, etc.) appear under some growth process. The process of accretion or deposition of new material to a solid

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is studied in the fundamental scientific area called Mechanics of Growing Solids. This area deals with all sorts of solid materials including elastic, viscoelastic, plastic, composite and graded materials (see, e.g., [1–7, 13]).

Additive manufacturing technologies are a particular case of growth processes. Mathematical modeling of additive manufacturing technologies is aimed at improving the performance of device, machine, and mechanism parts. The fundamentally new mathematical models considered in the paper describe the evolution of the end product stress-strain state in additive manufacturing and are of general interest for modern technologies in engineering, medicine, electronics industry, aerospace industry, and other fields (see, e.g., [8–12]).

By a (piecewise) continuously growing solid we mean a solid solid whose composition, mass or volume varies as a result of a (piecewise) continuous addition of material to its surface. The process of adding new material to the solid is called accretion or growth. For piecewise-continuous accretion the following basic stages of its deformation are strictly followed: before accretion, during the continuous growth, and after the accretion has ceased and growth has stopped. Each of these stages is characterized by the times when it starts and ends. The first is characterized by the time of application of a load to the solid and the time when growth starts. The second by the time when growth starts and the time when it ends. Conversely, the third is characterized by the time when growth ends and the time when it starts. The process under investigation is usually concluded by the third stage, for which the time when the next stage begins is taken to be as long as desired. The solid on whose surface new material is deposited starting from the time when accretion starts is called the basic or original solid. The solid consisting of the material pieces added to the basic solid over the time interval from the beginning of accretion up to a given instant of time is called the additional solid. The additional solid can have a complex structure and consist of a collection of solids formed over different time intervals of continuous accretion. We call them sub-solids. The additional solid is obviously the union of sub-solids. The domains occupied by the former and latter can be disconnected. The union of the basic and the additional solids will be called the accreted or growing solid. Note that accretion can also occur without the basic solid, starting from an infinitesimal material element. The part of the surface where infinitesimal pieces of the material are deposited at the actual instant is called the accretion or growth surface. The growth surface may be disconnected, in general. In particular, it can be the whole surface of the solid. Finally, the part of the surface of the original or the growing solid that coincides with the growth surface at the time when growth starts will be called the base surface. The base surface is clearly the part of the surface of the solid on which material is to be deposited during the next stage of continuous accretion. At different stages it coincides, as a rule, with the surface between the basic solid and the additional solid as well as with the surfaces between the sub-solids.

We assume that the basic solid, which is made from a viscoelastic ageing material (see, e.g., [13]), occupies a domain Ω_0 with the surface S_0 and is free of stresses up to the time τ_0 of application of the load. From τ_0 up to the time τ_1 when accretion starts the classical boundary conditions are given on S_0, the specific form of which is stated below. At τ_0 the continuous accretion of a solid begins due to the addition of material particles to the accretion surface $S^*(t)$. As it grows, the solid occupies a domain $\Omega(t)$ with surface $S(t)$. It is obvious that $S^*(t) \subseteq S(t)$. The time when a particle characterized by a position vector x is deposited on the solid will be denoted by $\tau^*(\mathbf{x})$ and called the time of deposition of the particle on the growing solid. The configuration of the accreted solid is completely defined by the function $\tau^*(\mathbf{x})$ depending on the spatial coordinates. Boundedness and piecewise-continuity are the general conditions usually imposed on $\tau^*(\mathbf{x})$.

We denote by $\tau_1^*(\mathbf{x})$ the time when an element of the growing solid is formed and by $\tau_0(\mathbf{x})$ the time when a load is applied to it. Naturally, $\tau_1^*(\mathbf{x}) \leq \tau_0(\mathbf{x}) = \tau_0$ for the elements of the basic solid ($\mathbf{x} \in \Omega_0$).

To simplify the problem we consider the case of small deformations and zero volumetric force.

2. Basic Concepts

We suggest an approach to modeling surface growth processes in solids on the basis of the following postulates:

- The surface growth of a solid is modeled by the motion of its boundary due to the influx of new material to the surface of the solid.
- The boundary conditions on the moving boundary (the growth surface) are found from an additional solid–surface contact interaction problem depending on the specific features of the growth process.