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A Robustness Generalized Distance Function Approach for Multiresponse Robust Optimization

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Abstract

Experimental design with multiple responses has received a great deal of attention from engineers, statisticians and experimenters recently. Generally, robustness and optimization have the same significance in the analysis of a statistical procedure. Compared with much research work on how to simultaneously optimize multiple responses based on certain criteria or objective functions, little research has been done on how robust the optimum solution is. The robustness in this paper refers to low sensitivity of the responses to the fluctuations of the input variables. On the basis of the proposal of a measure of robustness, this paper presents a robustness generalized distance function (RGDF) approach for multiresponse robust optimization, which modifies the generalized distance function (GDF) via a correction matrix. The proposed method takes into consideration of both robustness and optimization. It is illustrated with an example, which shows that the robustness generalized distance function approach gives more robust optimal condition on which multiple responses are simultaneously optimized and insensitive to small changes of input variables.

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1. Introduction

In many designed experimental situations, several responses are considered simultaneously. Multiresponse analysis has received a great deal of attention from engineers, statisticians and experimenters recently. Overviews of multiresponse surface methodology can be found in the publications by Khuri and Cornell [1], Myers, Montgomery

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and Vining et al. [2], Myers and Montgomery [3], Park [4], and Khuri and Valeroso [5]. One of the objectives in multiresponse analysis is the determination of a set of input variables for optimizing several responses simultaneously, which is called multiresponse optimization.

There have been many different approaches provided in the literatures for the multiresponse optimization problems. Lind, Goldin, and Hickman [6] proposed an overlaid contour plot approach which is a straightforward graphical method for optimizing multiple responses. However, this method can only deal with the problems with two or three design variables. When the number of the design variables is larger than three, this method becomes infeasible.

The desirability function approach (DFA) was first introduced by Harrington [7], and then improved and popularized by Derringer and Suich [8]. However, it assumes the responses are independent and does not take into consideration the variance-covariance structure of the responses. In practice, strong correlations among several responses often exist. Ignoring the correlation structure among the responses may lead to inaccurate optimum solutions. Kim and Lin [9] proposed an exponential desirability function which is robust to the potential dependences between response variables.

Khuri and Conlon [10] proposed the generalized distance function (GDF) approach for the simultaneous optimization of multiple responses that computes the generalized distance of the estimated responses to their respective optimum values. The optimum solution is found by minimizing the generalized distance. This approach can be considered as a generalization of the Mahalanobis distance. Pignatiello [11], Ames et al. [12], Vining [13], and Ko, Kim, and Jun [14] extended the generalized distance function approach and proposed several methods using the loss function approach based on a weighted squared error loss function. The major advantage of the generalized distance function approach is its consideration of the variance-covariance structure of the responses which is used in the analysis of correlative multiresponse optimization.

Compared with much research work on how to simultaneously optimize multiple responses based on certain criteria or objective functions, little research has been done on how robust the optimum solution is. Generally, robustness and optimization have the same significance in the analysis of a statistical procedure. In this paper robustness means that the responses are insensitive to the fluctuations of the input variables. The results are considered robust solutions if the responses are not affected much by small changes in the input variables. If a process is not robust, small changes of the input variables may result in big changes of the responses. With the consideration of robustness for the multiresponse optimization problems, a key issue is the choice of the robustness optimization criterion function. The generalized distance function approach is one of the most popular methods for the multiresponse optimization problem. Moreover, the generalized distance function takes into account the correlation in the data, since its calculation uses the variance-covariance matrix of the data set.

The purpose of this paper is to propose a robustness generalized distance function approach in which the optimization and the robustness are both under consideration. The next section provides a brief review of the generalized distance function approach. Then the measure of robustness is given, and a robustness generalized distance function approach is proposed, which modifies the generalized distance function via a correction matrix. Finally, the proposed method is illustrated with an example.

2. Generalized distance function approach

The generalized distance function approach was introduced by Khuri and Conlon [10]. Suppose that there are k variables $\{x_1, x_2, \ldots, x_k\}$ and m responses $\{y_1, y_2, \ldots, y_m\}$. Let n be the number of experimental runs and p be the number of regression parameters. It is assumed that all the m response models are of the same degree and form, and are fitted using the same design within the experimental region. The second-order model for the i_{th} response can be written in the vector form

$$\mathbf{y}_i = \mathbf{X}\boldsymbol{\beta}_i + \boldsymbol{\varepsilon}_i, \quad i = 1,2,\ldots,m \tag{1}$$

where $\mathbf{y}_i$ is a n×1 vector of observations on the i_{th} response, **X** is a n×p matrix whose each row is a function of the design settings for the corresponding experimental run, where $p = (k+1)(k+2)/2$, $\boldsymbol{\beta}_i$ is a p×1 column vector of the regression coefficients for the i_{th} response, $\boldsymbol{\varepsilon}_i$ is a n×1 vector of random errors associated with the i_{th} response,