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Projection method in material point method for modeling incompressible materials

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Abstract

Material point method is a variant of the finite element method (FEM) and is successfully applied in large deformation problems. Recently, material point method has been applied in a wide range of engineering applications including solid and solid-fluid interaction problems. However, describing the behavior of incompressible materials is a challenging problem in MPM. The explicit formulation and the linear elements used in the standard MPM exhibit numerical instabilities such as mesh locking and artificial pressure oscillations in material incompressibility. Further, the small time step used to obtain a reasonable numerical stability limits the application of MPM in problems particularly with long time durations. We present an implicit treatment of the pressure term in MPM to mitigate the numerical instabilities and small time steps in incompressible material problems. The set of velocity-pressure coupled governing equations resulted by the implicit formulation is solved using Choin's projection method. The numerical examples show that the present MPM implementation is capable of modeling incompressible materials without pressure oscillations using a significantly large time step.

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Keywords: material point method; incompressible materials; projection method.

1. Introduction

Material point method (MPM) which is categorized as a particle based method was first developed for solid mechanics problems with history dependent materials and large deformations. Recently, MPM has been used in many engineering applications including solid and solid-fluid interaction problems.

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However, the standard explicit formulation and the linear elements used in MPM lead to artificial pressure oscillations and significantly small time steps in incompressible and near incompressible material problems. The common approach to obtain a reasonable stable solution to the material incompressibility is to use enhanced strain values for pressure calculation [1,2]. Even though these enhancing techniques improve the pressure calculation, the standard explicit MPM still experiences small time steps and numerical instabilities particularly in complex material behaviors with a strong dependency on the pressure. The aim of our work is to present an improved MPM formulation for modeling incompressible materials.

We implement an implicit treatment of the pressure within MPM to model incompressible materials. Chorin's projection method is used to solve the velocity-pressure coupled governing equations resulted by the implicit formulation. Projection method was originally introduced by [3] to solve incompressible Navier-stokes equations in fluid mechanics applications. Later, the projection method has been extensively used in both mesh based methods [4, 5] and particle based methods [6, 7] to solve incompressible fluids. The use of the projection method in MPM is comparatively new. The work by [8] presents successful application of the Chorin's projection method in MPM to model arbitrarily incompressible and phase changing materials. In this paper, we study the effectiveness of the Chorin's projection method in MPM with linear quadrilateral elements for fully incompressible materials.

2. Formulation of MPM for incompressible materials

2.1. Governing equations

Let $\rho(\mathbf{x},t)$ be the density, $\mathbf{v}(\mathbf{x},t)$ be the velocity, $\boldsymbol{\sigma}(\mathbf{x},t)$ be the Cauchy stress tensor and $\mathbf{b}(\mathbf{x},t)$ be the body force of the material point in its current configuration. Neglecting thermal effects, the motion of the continuum body is governed by the conservation of mass, conservation of momentum and constitutive equations. Applying the incompressible condition and splitting the Cauchy stress tensor in to dilational part, p and deviatoric part, $\boldsymbol{\tau}$, the governing equations can be written in the following implicit form.

$$\rho\frac{\boldsymbol{v}^{t+1}-\boldsymbol{v}^{t}}{\Delta t} = -\nabla p^{t+1} + \nabla.\boldsymbol{\tau} + \rho\boldsymbol{b} \tag{1}$$

$$\nabla.\boldsymbol{v}^{t+1} = 0 \tag{2}$$

$$\boldsymbol{v}^{t+1} = \boldsymbol{v}_b \;\; on \;\; \partial\Omega_D \tag{3}$$

where $\partial\Omega_D$ is the Dirichlet boundary surface.

The coupled equations (1) and (2) are solved using the Chorin's projection method. In this technique, equation (1) is split into two equations (Eqs. (4) and (5)) introducing an intermediate velocity, $\boldsymbol{v}^*$ with appropriate boundary conditions (Eq. (6)).

$$\rho\frac{\boldsymbol{v}^{*}-\boldsymbol{v}^{t}}{\Delta t} = \nabla.\boldsymbol{\tau} + \rho\boldsymbol{b} \tag{4}$$

$$\rho\frac{\boldsymbol{v}^{t+1}-\boldsymbol{v}^{*}}{\Delta t} = -\nabla p^{t+1} \tag{5}$$

$$\boldsymbol{v}^{*} = \boldsymbol{v}_b^{*} \;\; on \;\; \partial\Omega_D \tag{6}$$

The implicit pressure term is solved by taking the divergence of equation (5) and substituting equation (2) in order to eliminate$\nabla.\boldsymbol{v}^{t+1}$. This results in an elliptic equation as

$$\nabla^2 p^{t+1} = \frac{\rho}{\Delta t}\nabla.\boldsymbol{v}^{*} \tag{7}$$