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A high order material point method

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Abstract

The classical material point method (MPM) developed in the 90s is known for drawbacks which affect the quality of results. The movement of material points from one element to another leads to non-physical oscillations known as 'grid crossing errors'. Furthermore, the use of material points as integration points renders a numerical quadrature rule of limited quality. Different solutions have been proposed in recent years to overcome these drawbacks. In this paper the approach of combining quadratic B-spline basis functions with a reconstruction based quadrature rule is pursued to solve these numerical problems. High-order B-spline basis functions solve the problem of grid crossing completely, whereas the considered reconstruction based quadrature rule reduces the quadrature error observed with MPM. In addition, the use of quadratic B-splines leads to a more accurate piecewise linear approximation of the stress field compared to the piecewise constant one obtained with linear Lagrangian basis functions commonly used with MPM. Two 1D benchmarks are considered involving large deformations, a vibrating bar and a column under self-weight. They render excellent results when adopting this high-order MPM.

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1. Introduction

The MPM is a hybrid particle-mesh method [1,2]. It combines a fixed background mesh with a set of material points moving through the grid which discretize a deforming continuum. The material points store all information of the continuum such as density, stresses and strains. The mesh is used to solve the equations of motion of the continuum every time step after their assemblage from material point data.
Over the years, MPM has been used successfully for the simulation of a wide variety of complex engineering problems involving large deformations and history-dependent material behaviour such as sea ice dynamics [4] and slope failure [9].
The equations of motion may be solved in a variational framework adopting the finite element method (FEM) with linear C^0 Lagrange finite elements. The discontinuity of their gradients across element boundaries leads however to non-physical oscillations when material points cross element boundaries. This phenomenon, known as 'grid crossing error', significantly affects the quality of the MPM solution and may even lead to a lack of spatial convergence [5].

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Different measures have been adopted as a remedy such as GIMP [6], the Dual Domain Material Point (DDMP) method [7] and mixed-integration approaches [8,9].

A further limitation to accuracy lies in the numerical integration. Material points are used as quadrature points and their volumes are interpreted as quadrature weights. The resulting numerical quadrature rule can be of very poor quality once material points become arbitrarily distributed. The use of standard quadrature rules such as Gaussian rules is not straightforward since physical quantities such as stresses and strains are only known at material point positions. Reconstruction techniques like the Moving Least Squares (MLS) approach [10] or interpolation as in [8,9] can be used to extend the discrete particle data to functions and apply standard quadrature rules on these to reduce the quadrature error of the MPM-type integration.

In this paper, a novel spline-based approach is proposed that combines high-order B-spline basis functions for the approximate solution of the equations of motion on the background grid with the improved accuracy of reconstruction-based numerical quadrature based on cubic spline interpolation.

Quadratic B-spline basis functions have continuous first derivatives which eliminate the effect of grid crossing errors entirely. Cubic spline interpolation of the discrete material point data and numerical integration by a sufficiently high Gauss quadrature rule reduces the integration error significantly. The performance of the novel MPM approach is demonstrated with 1D large deformation benchmarks.

In the following section, the devised spline-based MPM is described. Results obtained with spline-based MPM for different benchmarks are presented in Section 3. Conclusions are drawn in Section 4.

2. Spline-based MPM

In essence, the modifications of this approach are limited to the spatial discretization of the equations of motion on the background grid and the way in which integrals are evaluated numerically. The general MPM procedure has not been altered. It follows the formulation presented in [9] or [11] to which the reader is referred for a detailed description of MPM. Time integration is performed by the semi-explicit Euler-Cromer scheme [9].

2.1. Space discretization

B-spline basis functions of order d are uniquely defined by an underlying *knot vector* $\Xi = \{\xi_1, \xi_2, \ldots, \xi_{n+d+1}\}$ consisting of a set of non-decreasing *knots* ξ_i. A knot vector of length $n + d + 1$ defines n basis functions of order d. Constant B-spline basis functions are defined as:

$$\phi_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \leq \xi < \xi_{i+1} \\ 0 & \text{else.} \end{cases}$$

Higher-order B-spline basis functions are then defined by the Cox-de Boor recursion formula [12]

$$\phi_{i,d}(\xi) = \frac{\xi - \xi_i}{\xi_{i+d} - \xi_i}\phi_{i,d-1}(\xi) + \frac{\xi_{i+d+1} - \xi}{\xi_{i+d+1} - \xi_{i+1}}\phi_{i+1,d-1}(\xi),$$

where $\xi \in [\xi_1, \xi_{n+d+1}]$. The non-empty intervals $[\xi_i, \xi_{i+1})$ are called *knot spans*. An open uniform knot vector is used, i.e. the knots are equally distributed and the first and last knots are repeated d times. Given an open uniform knot vector of length $n + d + 1$, the number of knot spans is equal to $n - d$. Quadratic B-spline basis functions are adopted. An example of quadratic B-spline basis functions derived from an open uniform knot vector is depicted in Figure 1.