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Post-buckling behaviour of axially FGM planar beams and frames

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Abstract

Post-buckling analysis of planar beams and frames made of axially Functionally Graded Material (FGM) by using the finite element method is presented. The material property of the beams is assumed to vary linearly along the axis of beam direction. A non-linear beam element based on Timoshenko beam theory is formulated in the context of the co-rotational formulation. The non-linear equilibrium equations are solved by using the incremental/iterative procedure in combination with the arc-length control method. The obtained results are compared with the published references to verify the accuracy of the proposed formulation and the numerical procedure. The effect of the material distribution on the post-buckling response of the axially FGM structures is highlighted.

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Keywords: Axially FGM; Large displacement analysis; Planar beam and frame structures; Finite element method

1. Introduction

The post-buckling behaviour of planar beams and frames is very important information for design engineers. Experiments on buckling of long and short cylinder beam can be found in [1]. Establishing equilibrium paths is the most common way of understanding the structures' behaviour in the post-buckling state. However, a major difficulty is the geometric nonlinearity when the structures undergo large displacement. Due to this challenge, it is difficult to analyse the post-buckling problem using analytical methods, and a numerical method, especially the finite element method, is often employed instead. In order to analyse the large displacements of the structures by the finite element method, a nonlinear finite element which makes it possible to model the nonlinear behaviour of the structures

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accurately is required. Many nonlinear finite elements in general and beam elements in particular are available in the literature, some of which have been documented in well-known textbooks [2,3]. The nonlinear beam elements can be classified into three types, namely the total Lagrange formulation, the updated Lagrange formulation, and the co-rotational formulation [4]. In the co-rotational formulation which will be adopted in this article, the kinematics is described in an element-attached local coordinate system. The finite element formulation is firstly formulated in this local system and then transferred to a global system with the aid of transformation matrices. Among others, the elements proposed by Hsiao and Huo [5], Meek and Xue [6], Pacoste and Eriksson [7], and Nguyen [8] are some of the co-rotational beam elements for analysis of planar beams and frames which can be cited herewith.

Functionally graded material (FGM) has received great interest from many researchers and engineers for a long time because of its wide range of applications in structural mechanics. FGMs, which were introduced by Japanese scientists in 1984, can be formed by varying the percentage of constituents in any desired direction in order to create new materials with specific physical and mechanical properties [9]. The newly created materials have then been employed in many fields such as space projects, energy sectors, communication projects, defense industries, biomedicine, and miscellaneous others. Many investigations of FGM structures are available in the literature, and only contributions that are most relevant to the present work are discussed below. Trinh and Gan [10] developed a consistent shape function for a linearly solid tapered Timoshenko beam. Shahba et al. [11,12] employed the exact shape functions from a uniform homogenous Timoshenko beam segment to formulate a finite element formulation for computing natural frequencies and buckling loads of tapered axially FGM beams. Nguyen [13], Nguyen and Gan [14], and Nguyen et al. [15,16] derived the finite element formulation for studying the large displacement behaviour of FGM beams and frames. From the literature review, there exist many investigations of FGM beams and frames in the thickness direction; however, researches on the post-buckling analysis (with large displacement taken into account) of axially FGM structures are still limited. This paper aims to contribute research on the geometric nonlinearity in analysing the post-buckling behavior of axially FGM structures to the existing literature.

The present article investigates the post-buckling response of planar beams and frames made of axially FGM by using the finite element method. The material properties of the structures are assumed to vary linearly in the axial direction. A shear deformable nonlinear beam element, taking into account the effect of the material non-homogeneity, is formulated in the context of the co-rotational formulation. A consistent formulation of shape functions for a Timoshenko beam section made of axially FGM is derived based on the Hamilton principle. An incremental/iterative procedure in combination with the arc-length control method is employed to compute equilibrium paths. Numerical examples are given to demonstrate the accuracy and efficiency of the formulated element and to examine the effect of the material non-homogeneity on the instability behaviour of the structures.

2. Consistent shape functions for axially FGM beam element

2.1. Problem definition

In the Bernoulli-Euler beam, the cross-section of the beam is assumed to remain straight against the axis after it has deformed; hence no shear deformation is taken into account. Hence, the resulting shape functions contain only the length information of the beam inside the polynomial forms. For a short beam where the shearing deformation is a dominant factor for getting an accurate result, the Bernoulli-Euler beam is not recommended. Therefore, the Timoshenko beam has to be considered.

Reference [17] derived the shape functions for the Timoshenko beam (Fig. 1) by using Hamilton's principle to satisfy the homogeneous form of the static equations of equilibrium as

$$\frac{\partial}{\partial x}\left(EA\frac{\partial u}{\partial x}\right)=0, \qquad \frac{\partial}{\partial x}\left(\kappa AG\left(\frac{\partial w}{\partial x}-\theta\right)\right)=0, \qquad \frac{\partial}{\partial x}\left(EI\frac{\partial \theta}{\partial x}\right)+\kappa AG\left(\frac{\partial w}{\partial x}-\theta\right)=0. \tag{1}$$

where A and I are the beam cross-section of the area and the area moment of inertia, respectively, E and G are the Young's and shear moduli of the beam material, respectively, and κ is the shear coefficient of the cross-section