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Development of the DKMQ element for buckling analysis of shear-deformable plate bending

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Abstract

In this paper the discrete-Kirchhoff Mindlin quadrilateral (DKMQ) element was developed for buckling analysis of plate bending including the shear deformation. In this development the potential energy corresponding to membrane stresses was incorporated in the Hu-Washizu functional. The bilinear approximations for the deflection and normal rotations were used for the membrane stress term in the functional, while the approximations for the remaining terms remain the same as in static analysis. Numerical tests showed that the element has good predictive capability both for thin and thick plates.

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Keywords: Shear Deformable Plate Bending;DKMQ; Buckling Analysis; Kirchhoff; Reissner-Mindlin

1. Introduction

Plate bending is of an utmost important structural model in engineering. To analyze practical problems of plate bending, the finite element method (FEM) is at present the most widely used numerical method. Indeed the plate bending problem is one of the earliest problems to which the FEM was applied [1]. The most commonly used theories in developing finite elements for analysis of plate bending are Kirchhoff (or thin plate) and Reissner-Mindlin (or thick plate) theories. The Kirchhoff plate theory neglects the effect of shear deformation and thus it is only valid for thin plates, whereas the Reissner-Mindlin (RM) plate theory is applicable to both thick and thin plates. In early development of the FEM, the Kirchhoff theory was widely adopted as the basis of the finite element

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formulation. The difficulty with this approach is to construct the shape functions that satisfy the C^1 continuity requirement. In the subsequent developments the RM theory is preferred since it requires only C^0 continuity on the shape functions and furthermore, it is a more general theory than the Kirchhoff theory.

While the use of RM theory in developing plate elements by-passes the difficulty caused by the C^1 requirement, direct application of the displacement-based finite element formulation, however, produces elements that overly stiff for thin plate situations. This phenomenon is known as shear locking. Early attempts to overcome this difficulty wasto employ the selective reduced integration technique (e.g. see [2] and the references therein). Unfortunately this simple approach produced elements that have spurious energy modes. Since then there are innumerable RM plate bending elements have been proposed with different approaches to eliminate the shear locking. Some recently proposed successful plate bending elements include the refined Mindlin plate elements [3,4], a family of RM plate elements formulated using the discrete shear gap concept [5,6], and the RM plate element based on the consistent version of the Mindlin equations [7].

Among countless plate bending elements available now, the discrete-Kirchhoff Mindlin quadrilateral (DKMQ) element proposed by Katili [8] is of our interest since it has the standard nodal degrees of freedom, pass the patch test, shear locking free, and no spurious zero energy modes. Furthermore it has been proven [8] that the DKMQ has good predictive capability for thin to thick plates. This element is an extension of the DKQ (discrete Kirchhoff quadrilateral) element [9], which is a simple, efficient and reliable element for analysis of thin plates to include the shear deformation. The DKMQ [8] results will converge to the DKQ [9] results as the plate becomes progressively thinner.

With regard to the good characteristics of the DKMQ element, this element has been recently further developed to the DKMQ24 shell element [10] and applied to composite plate bending structures [11,12]. However, to the authors' knowledge, there is no published report on the application of the DKMQ to plate bending buckling problems. It is thus the aim of this paper to present the development of the DKMQ element to plate buckling problems.

In the present development the membrane strain energy was added to the original Hu-Washizu functional for RM plates in order to account for the membrane stress effect to the plate bending stiffness. The approximate deflection and rotation fields for the membrane strain energy were taken to be the standard bilinear function, while the approximate fields for the bending and shear strain energy followed the original work [8]. The element was tested to different plate buckling problems to assess the accuracy and convergence characteristics. The results showed the DKMQ element can give accurate critical bucking loads both for thin and thick plates.

2. Formulation of the DKMQ for bucking analysis

A detailed formulation of the DKMQ for static analysis of plate bending have been presented in Reference [8]. In this section we present only the essential equations of the static formulation. The focus is given to formulation of the DKMQ for buckling problems.

2.1. Variational formulation

We consider a plate of uniform thickness h, made from homogeneous and isotropic material with modulus of elasticity E and Poisson's ratio v. Three dimensional Cartesian coordinate system is established with the x-y plane lying on the plate middle surface A as illustrated in Fig 1. Based on basic assumptions of the RM plate theory, the displacement of a generic point in the plate can be expressed as

$$u = z\beta_x(x, y), \quad v = z\beta_y(x, y), \quad w = w(x, y) \tag{1}$$

Where w is the deflection of the middle surface A, β_x and β_y are the normal line rotations in the x-z and y-z planes, respectively.

The strains associated with bending deformation, $\langle\varepsilon_b\rangle$, can be expressed in terms of the curvature, $\langle\chi\rangle$, as

$$\langle\varepsilon_b\rangle = z\langle\chi\rangle, \ \ \langle\chi\rangle = \langle\frac{\partial\beta_x}{\partial x}\frac{\partial\beta_y}{\partial y} \quad (\frac{\partial\beta_x}{\partial y}+\frac{\partial\beta_y}{\partial x})\rangle \tag{2a, b}$$