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Mesh sensitivity in peridynamic quasi-static simulations

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Abstract

Peridynamics is a non-local formulation of continuum mechanics that does not rely on spatial derivatives, therefore peridynamics is well suited for crack and failure modeling. Body is discretized in a finite number of particles and each particle connects to other particles within a range called a material's horizon. In this study authors performed fifty glass-fiber coupon tensile and compression simulations with different horizon size and particle spacing combinations to see how they influence maximum displacement. Values from simulations are compared with values calculated using Hooke's law. The results show that the horizon size of three particle spacings gives the best results, which, support's the general view on the issue. However, simulations with horizon sizes $\sqrt{2}$ times larger than particle spacing show similar accuracy.

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1. Introduction

Peridynamic theory [1] is a nonlocal formulation of continuum mechanics that was created to handle materials' discontinuities such as cracks. Classical theory relies on spatial derivatives to represent the relative displacement and force between two particles and partial derivatives with respect to the spatial coordinates are undefined along the discontinuities. In contrast, peridynamics (PD) uses integral equations that do not require spatial derivatives. PD was first introduced in the bond-based form [2], in which Poisson's ratio is limited to 0.25. In 2007 Silling et al. [3] introduced the state-based formulation that eliminated this restriction. PD body is discretized in a number of particles each describing some amount of volume and the side of a particle is commonly called a lattice. The results depend on

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particle positions, spacing h, and material's horizon δ (see chapter 2 for PD theory). For three-dimensional analysis the most common are cubic lattices with uniform spacing in all directions [4–8]. PD have been used successfully to predict damage and failure [8–12], however, few articles deal with the mesh spacing's and horizon's influence on the results.

Given a domain of m discretized particles with h being the distance between two neighboring particles, Bobaru et al. defined three convergence types in [13]:

- h convergence, where δ is fixed as $h \to 0$;
- (δh) convergence, where δ decreases and h decreases, but the ratio between them stays the same;
- δ convergence, where $\delta \to 0$ with a fixed h.

Yaghoobi and Chorzepa considered (δh) and δ convergence types for a two-dimensional mesh in [6] and found that the results are the most accurate with $\delta/h = 3.015$. In [4] Silling and Askari used four mesh spacings to show how crack growth changes based on the mesh spacing in a two-dimensional plate. Study [14] showed that in two-dimensional plate with a crack the difference in displacements between PD and finite-element method (FEM) shrinks as δ decreases. Henke and Shanbhag [15] found that cubic mesh with $\delta/h = 4$ gives better results than meshes with lower δ/h value, however, centroidal Voronoi tessellation mesh gives similar or better results. In [16] δ convergence and h convergence studies were performed for unidirectional composites. Previously mentioned studies (with the exception of [15]) used two-dimensional models for their analysis.

For this study, authors simulated a tensile and a compression test of glass-fiber coupons. Simulated displacement values are compared with displacement values obtained from Hooke's law and percent errors are calculated. Different h and δ values to study five cases of δ and five cases of (δh) convergence were used.

2. Peridynamic theory

In PD an undeformed body consists of an infinite number of particles identified by their coordinates, $\mathbf{x}_{(i)}$. Each particle is associated with some amount of volume $V_{(i)}$. These particles undergo displacement $\mathbf{u}_{(i)}$ and their position in the deformed configuration is described by the position vector $\mathbf{y}_{(i)}$. Each particle has a range $\delta > 0$ called the "horizon", named so because the particle cannot "see" past it. In three-dimensional space particles within a sphere, with a radius of δ and centered at particle $\mathbf{x}_{(i)}$, are called the family of $\mathbf{x}_{(i)}$, $H_{\mathbf{x}_{(i)}}$. An example of peridynamic body is shown in Fig. 1. Particle $\mathbf{x}_{(i)}$ interacts through bonds $\langle \mathbf{x}_{(j)} - \mathbf{x}_{(i)} \rangle$ with all particles in its family and the bond properties depend on material models. The force density vector, which can be viewed as the force exerted by particle $\mathbf{x}_{(i)}$ on the particle $\mathbf{x}_{(j)}$, is then $\mathbf{t}_{(i)(j)}$. Similarly, particle $\mathbf{x}_{(i)}$ is influenced by $\mathbf{x}_{(j)}$ through the force density vector $\mathbf{t}_{(j)(i)}$. These forces are determined jointly by the collective deformation of neighborhoods $H_{\mathbf{x}_{(i)}}$ and $H_{\mathbf{x}_{(j)}}$ throughout the model. Force density vectors $\mathbf{t}_{(i)(j)}$ where $(j = 1, 2, ..., \infty)$ associated with particle $\mathbf{x}_{(i)}$ are stored in infinite-dimensional array, called a force vector state, $\underline{\mathbf{T}}$ (1) and the relative position vectors in the deformed configuration $(\mathbf{y}_{(j)} - \mathbf{y}_{(i)})$ where $(j = 1, 2, ..., \infty)$ can be stored in an similar array called a deformation vector state, $\underline{\mathbf{Y}}$ (2).

$$\underline{\mathbf{T}}(\mathbf{x}_{(i)}, t) = \begin{Bmatrix} \mathbf{t}_{(i)(1)} \\ \vdots \\ \mathbf{t}_{(i)(\infty)} \end{Bmatrix}, \tag{1}$$

$$\underline{\mathbf{Y}}(\mathbf{x}_{(i)}, t) = \begin{Bmatrix} (\mathbf{y}_{(1)} - \mathbf{y}_{(i)}) \\ \vdots \\ (\mathbf{y}_{(\infty)} - \mathbf{y}_{(i)}) \end{Bmatrix} \tag{2}$$

Force vector state for particle $\mathbf{x}_{(i)}$ depends on the relative displacements between that particle and all other particles within its horizon, therefore force vector state can also be written as