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Admittance/Fourier series revisited: understanding periodic heat flows

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Abstract

Thermal modelling of buildings typically involves the use of software programs that are highly accurate but complex. As such many users do not have a good "feel" for how heat flows in and out of a building. The simplest type of manual calculation method is a steady state model which allows some insight into the flow of heat in a building. However modelling of thermal storage in building elements with mass is seen to be too difficult to be solved readily and as such complex thermal software programs are utilised. In the late 1960s and early 1970s the admittance method was developed which calculated quite accurately the thermal response of building elements with mass. Typically the thermal response of a building to 24 hour cyclic inputs - temperatures and solar radiation – was calculated. When extended to higher frequencies utilising a Fourier series representation of temperatures and solar radiation, accuracy improves further. However this approach was soon overtaken by more complex computer based models which delivered greater modelling complexity and accuracy but tended to obscure the underlying physical processes. This paper re-examines the admittance/Fourier method as a pathway to enhancing understanding of the response of buildings to fluctuating temperatures and solar radiation. A simplified representation of yearly ambient temperature in terms of only three terms: a constant, a yearly and daily frequency – allows a very simple model of a building to be developed. This approach can allow building designers rapid insight into the performance of various materials and designs and in addition enhance their understanding of the fundamental physical processes involved.

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1. Introduction

The analysis of heat flowing in and out of a building can be carried out in a number of ways. Starting with simple steady state analysis and utilizing the thermal resistance of building elements (or their inverse, *U*-values) – steady state calculations allow some insight into the thermal performance of a building design. However once time varying external temperatures, solar radiation and building elements with thermal capacitance need to be analyzed most researchers and practitioners utilize a sophisticated, modern computer based thermal analysis program to tackle the problem. Of course this latter approach has been shown to be accurate in calculating a wide range of heat transfer processes that are complex and necessary in designing buildings. However it seems that there is little understanding of the middle ground between these two cases of a simple steady state calculation and a complex, thermal computer based program. As such inexperienced designers, practitioners and researchers are often at the mercy of accepting whatever results a complex thermal program produces. On the other hand those with many years' experience may have a better physical "feel" for these complex calculations and be able to avoid situations where there are errors in the results they are obtaining perhaps due to errors in their input parameters or if there is an error in the program itself.

This paper seeks to revisit techniques for the analysis of periodic and time varying heat flows that were in vogue before complex computer calculations took over the field. The admittance approach was strongly advanced by researchers predominantly in the UK, such as Milbank and Harrington [1], Davies [2], as well as Athienitis and Santamouris [3], and Muncey [4]. It is not the intent of this paper to develop yet another approach to try and compete with existing complex thermal programs. Instead this paper seeks to elucidate and explore the admittance method of analyzing periodic heat flow in order to yield greater insight or "feel" for how time-varying heat flows actually occur through building elements for simplified cases.

1.1. Periodic solution of the heat equation

To analyse periodic heat flow in a building involves analysing the response of the building envelope to sinusoidal temperatures or irradiances. Due to Fourier's theory – any time varying function can be represented by a summation of different frequency cosine and sine waves [3]. Hence, any time varying temperatures, solar irradiance, mechanical heating or cooling, or any other heat flow can be analysed. . It is useful to define for any temperature, T as being equal to the sum of the steady state temperature, $\bar{T}$ and the periodic component $\tilde{T}$. That is, $T = \bar{T} + \tilde{T}$. Similarly for heat flows, $\dot{Q} = \bar{\dot{Q}} + \tilde{\dot{Q}}$.

Consider now a uniform wall of thickness L, density ρ, thermal conductivity k and specific heat capacity c_p. The periodic solution of the heat equation [5] as a function of distance x and time t, for the periodic temperature inside the wall $\tilde{T}(x,t)$ is the real part of:

$$\tilde{T}(x,t) = \tilde{T}_o(t)\,\frac{\cosh\big(\gamma(L-x)\big)+\frac{h_i}{k\gamma}\sinh\big(\gamma(L-x)\big)}{\left(1+\frac{h_i}{h_o}\right)\cosh(\gamma L)+\left(\frac{h_i}{k\gamma}+\frac{k\gamma}{h_o}\right)\sinh(\gamma L)}, \tag{1}$$

where $\tilde{T}_o(t)$ is the sinusoidal, outside temperature (real part of $|T_o|e^{j\omega t}$), $|T_o|$ is the amplitude of the outside temperature, and the inside temperature $\tilde{T}_i$ is set to zero. The terms h_i and h_o are the convection heat transfer coefficients at the inside and outside surface of the wall respectively, and γ is given by:

$$\gamma = \sqrt{\frac{\omega\rho c_p}{2k}}\,(1+j), \tag{2}$$

where ω is the angular frequency of the outside temperature (and is related to the period, P of the oscillation by the equation $P = 2\pi/\omega$), $j = \sqrt{-1}$, and hence γ is a complex quantity. Another useful quantity to define is the effective thickness, δ [5]:

$$\delta = \sqrt{2k/\omega\rho c_p}. \tag{3}$$