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Short Steel Thin-walled Columns Subjected to Eccentric Axial Loads

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Abstract

A simple analytic solution for the stresses and displacements of short thin-walled columns of open bisymmetric cross-sections subjected to eccentric axial loads is proposed. The theory of bending and torsion with influence of shear is applied. An analytic closed form solutions are obtained both for the stresses and displacements. The normal stresses are expressed by the compression component, the bending components in two principal planes and the torsion component with respect to the principal pole (the shear centre). The displacements are expressed with respect to the principal axes and the principal pole. The deflections and the angle of torsion are given by the bending and torsion components according to the classical theories of bending and torsion [8] and the additional components due to shear in bending and torsion according to the theory of bending and torsion with influence of shear [4, 6]. The results are compared and discussed to the results of the finite element analyses by using shell elements.

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Keywords: Columns, thin-walled, compression, bending, torsion, shear;

1. Introduction

Determination of stresses and displacements in relatively short thin-walled beams with open section subjected to bending and torsion, where the influence of shear must be taken into account both in bending and torsion, is the subject of newer investigations [2, 3, 6, 7]. In the case of very short thin-walled beams the Saint-Venant component of torsion may be neglected with respect to the warping component. In that case, there is a total analogy between bending and

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torsion [4]. In this paper, the theory of short thin-walled beams of open section subjected to tension, bending and torsion with influence of shear, when the Saint-Venant component may be neglected, will be applied in the analysis of short steel thin-walled columns of open bisymmetrical cross-sections subjected to eccentric axial loads. A comparison with the finite element analysis will be provided.

2. Stresses and Displacements

2.1. Stresses and Internal Force Components

The normal and shear stresses of thin-walled columns with open section in the case of eccentric loads are given as [4, 6, 8]

$$\begin{bmatrix}\sigma_x^u\\ \sigma_x^v\\ \sigma_x^w\\ \sigma_x^\alpha\end{bmatrix}=\begin{bmatrix}1&0&0&0\\0&y&0&0\\0&0&z&0\\0&0&0&\omega\end{bmatrix}\begin{bmatrix}N/A\\ M_z/I_z\\ M_y/I_y\\ B/I_\omega\end{bmatrix},\ \tau_{x\xi}^v=\frac{Q_yS_z^*}{I_zt},\ \tau_{x\xi}^w=\frac{Q_zS_y^*}{I_yt},\ \tau_{x\xi}^\alpha=\frac{M_\omega S_\omega^*}{I_\omega t} \tag{1}$$

where

$$I_z=\int_A y^2\,\mathrm{d}A, I_y=\int_A z^2\,\mathrm{d}A, I_\omega=\int_A \omega^2\,\mathrm{d}A, S_z^*=\int_{S^*} y\mathrm{d}A^*, S_y^*=\int_{S^*} z\,\mathrm{d}A^*, S_\omega^*=\int_{S^*}\omega\,\mathrm{d}A^*, dA=tds, dA^*=tds^*;$$

$N = N(x)$ is the normal force, $M_y = M_y(x)$ and $M_z = M_z(x)$ are the bending moments with respect the y and z-axis, $B = B(x)$ is the bimoment, $Q_z = Q_z(x)$ and $Q_y = Q_y(x)$ are the shear forces with respect the z and y-axes, $M_\omega = M_\omega(x)$ is the moment of warping, $y = y(s)$ and $z = z(s)$ are the rectangular coordinates, with respect to the principal cross-section axes C_y and C_z, $\omega = \omega(s)$ is sectorial coordinate with respect to the principal pole P (the shear centre) and principal starting point M, $t = t(s)$ is the wall thickness, C is the cross-section centroid, P is the cross-section pole, s is the curvilinear coordinate of the arbitrary point S from the starting point M, s^* is the curvilinear coordinate from the free edge, where the shear stress is equal to zero, ξ is the tangent through the point S (Fig. 1).

It is assumed that the Saint-Venant component of torsion may be ignored with respect to the warping component, i.e. that $M_p = M_\omega$, where M_p is the total moment of torsion, with respect to the cross-section principal pole P. It is shown that the Saint-Venant component of torsion can be neglected for small ratios of beam length with respect the cross-section contour dimensions; for instance for bisymmetrical cross-section for $l/h = 5$, $t/h = 40$, where l is the beam length, h is the section height and $t = const$, this simplification introduces a maximum error of 1.5%.

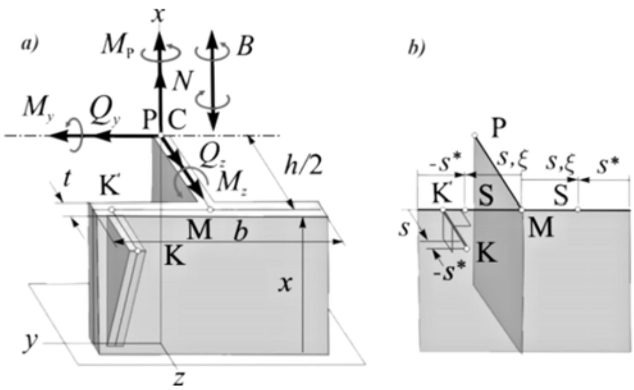

Fig. 1. Internal force components of the thin-walled columns of open bisymmetrical section subjected to eccentric loads.
a) half of the section; b) half of the middle surface.