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Modal analysis of thin aluminium plate

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Abstract

The article deal with modal analysis, like natural frequencies and own mode shapes of thin plate. In the first part of article the theory of modal analysis is given. Next part of article is a modal analysis of thin aluminium (Al 99.9) isotropic plate with dimensions 0.10 × 0.10 × 0.002 m. The plate is fixed over the circumference. The solution was carried by numerically in the ANSYS program. In the article the first ten natural frequencies and first ten mode shapes of the plate deflection are presented.

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Keywords: modal analysis; mode shapes; eigen frequency; thin plate;

1. Introduction

Modal analysis of oscillating systems determine eigenmodes and eigenvalues. These values can be determined by the simulation (eigenvalues) or experiment (natural frequencies and mode shapes). The eigen modes and the eigenvalues are used to investigate the vibration of mechanical structures, to diagnose architectural and engineering constructions. It is one of the basic dynamics methods. The principle of modal analysis is based on the possibility of decomposition of oscillatory motion for partial (also modal, own) parts. The resulting motion is created by superposing of these parts. Each part of the oscillating movement is characterized by its own frequency, eigen modes and the corresponding damping mode shape. A complete description of the dynamic mechanical system is obtained by determining the modal properties of the resulting parts.

The aim of the modal analysis is to find the natural frequencies and mode shapes of the system (parts). Modal, harmonic analysis is one of the dynamic analyzes, at solutions we are considering the inertia of the system. A mechanical system with one degree of freedom is presented in Fig.

This system is characterized by its own weight m, stiffness k, damping b and excitation force $F(t)$. The equation of motion of the system on Fig. 1 can be written as:

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$$m\ddot{x}(t) + b\dot{x}(t) + kx(t) = F(t) \tag{1}$$

where x – deflection [m], $\dot{x}$ – speed [m/s], $\ddot{x}$ – acceleration [m/s2], F – excitation force [N].

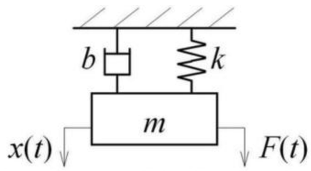

Fig. 1. Scheme of the system with one degree of freedom.

If we neglect the damping the equation (1) changes to form

$$m\ddot{x}(t) + kx(t) = F(t) \tag{2}$$

For solve the above-mentioned differential equations motion we introduce a boundary conditions $x(0) = x_0$; $\dot{x}(0) = \dot{x}_0$, $F(t) = 0$. The equation (2) will have the form

$$\ddot{x}(t) + \Omega_0^2\, x(t) = 0 \tag{3}$$

where

$$\Omega_0{}^2 = \frac{k}{m}. \tag{4}$$

Solution of equation (3) is in the form

$$x(t) = C\sin(\Omega_0 t + \varphi) \tag{5}$$

where C – amplitude [m], Ω – own angular frequency [rad/s], φ – phase angle [rad].

The equation (6) is oscillation amplitude and the equation (7) is phase angle.

$$C = \sqrt{\left(\frac{\dot{x}_0}{\Omega_0}\right)^2 + x_0^2} \tag{6}$$

$$\varphi = \mathrm{arctg}\frac{x_0\Omega_0}{\dot{x}_0}. \tag{7}$$

The system oscillates with angular frequency Ω_0. This angular frequency is called the natural frequency of the system. Each natural frequency of the system corresponds to one's own mode shape. Equation (3) can be written in matrix form

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = 0 \tag{8}$$

where $\mathbf{M}$ – mass matrix [kg], $\mathbf{K}$ – stiffness matrix [N/m], $\mathbf{x}(t)$ – displacement vector [m], $\ddot{\mathbf{x}}(t)$ – vector of