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Vibration of mechanical system with higher degrees of freedom: solution of the frequency equations

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Abstract

The solution of the motion equation of the rigid body systems with higher degrees of freedom $2 \leq p° \leq 10$ is difficult. The presented method allows solving the motion equations of such systems by its transformation to higher degrees' algebraic characteristic equations. The vibration of the system is then described by frequencies obtained from solution of characteristic equations. The proposed method follows Bezout's factor theorem, Bairstow-Hitchcock's method, method of synthetic division and other presuppositions given in the article. The solution is based on the determination of the complex zeros of polynomials.

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1. Introduction

The equations of motion are fundamental of investigation, which are aimed to vibration of the rigid bodies that could be resiliently supported and coupled, even with dissipative elements. The basic models of road and railway vehicles and some simple-operational machines, etc. are presented as examples of rigid bodies systems with lower degrees of freedom $p^{\circ} \leq 10$ approximately. We assume linear coupling of elastic even dissipative elements and small changes of displacement of system bodies. Then we arise to the system of ordinary second order differential linear inhomogeneous equations with constant coefficients, which could be written in matrix form as:

for non-damped vibration

$$\mathbf{M}\ddot{y} + \mathbf{K}y = F(t) \tag{1}$$

for damped vibration respectively

$$\mathbf{M}\ddot{y} + \mathbf{B}\dot{y} + \mathbf{K}y = F(t) \tag{2}$$

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where **M** is mass (inertia) matrix of the system, **K** is stiffness matrix, **B** is damping matrix, *F(t)* is vector of generalized excitation function and *y(t)* is vector of generalized coordinate. The analytical solution could be obtained by Lagrange method of constant variation or by application of integral transformation for low range system. When we applied Laplace's integral transformation for reverse image transformation (using convolution) it is necessary to determine the zeros of frequency polynomial:

for non-damped vibration from (1)

$$A_n(x) = \sum_{j=0,2}^{n} a_{n-j} . x^{n-j} \tag{3}$$

for damped vibration from (2)

$$B_n(x) = \sum_{j=0,1}^{n} b_{n-j} . x^{n-j} \tag{4}$$

i.e., to determine roots of frequency characteristic algebraic $n = 2p$ order equation

$$\sum_{j=0,2}^{n} a_{n-j} . x^{n-j} = 0\,, \quad \sum_{j=0,1}^{n} b_{n-j} . x^{n-j} = 0 \tag{5}$$

The real positive coefficients of polynomial (3) and (4) defined also by equation (5), a_{n-j} and b_{n-j} are the functions of matrix elements **M** and **K**, or **M**, **B**, **K** respectively. We consider standardized form of the frequency equations which means $a_n = 1$, $b_n = 1$. At first we determine the roots of the equations (5) by analytical solution of equation system (1) and (2). The extensive collection of approximate and numerical solution method of algebraic high order equations exists [6].

The specific properties of polynomial for vibration of mechanical systems of rigid bodies influence the choice of any suitable methods.

1) Polynomial (3) and (4), or equations (5) are of even order $n = 2p°$, where only even exponents of variable x are non-zero in equations for non-damped vibration and coefficients of odd exponents are generally equal to zero. The coefficients of even and odd variable exponents are generally non-zero for damping vibration.
2) The coefficients have explicit relation with each other in equations (5): the coefficients of polynomial (3) are components of coefficients of polynomial (4) see [1].
3) The roots of frequency equation are purely imaginary conjugated for non-damped vibration.

$$x_{k,k+1} = \pm i\omega_{0k} \; for\; k = 1,\, 3, \ldots,\, n\, -1 \tag{6}$$

The roots of frequency equation are complex conjugated for damped vibration,

$$x_{k,k+1} = -\beta_j \pm i\omega_j \; for \quad \mathrm{k} = 1,\, 3, \ldots,\, \mathrm{n} - 1 \qquad j = \frac{k+1}{2} = 1,2,\ldots,\, p \tag{7}$$

where ω_0 is inherent circular frequency of non-damping vibration, ω_j is inherent circular frequency with viscous linear damping less than critical, β_j is damping constant. The relation of these variables could be expressed:

$$\omega_k^2 = \omega_{0k}^2 - \beta_k^2 \tag{8}$$

We assumed only the simple roots x_k of equations (5) for $x = x_k \Rightarrow f'(x_k) \neq 0.$

4) The products of rooted factor of conjugated roots according to (6), (7) and (8) define quadratic polynomial for non-damped vibration.