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An influence of the longitudinal spring element on free vibrations of the partially tensioned column

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Abstract

In this paper the slender system subjected to Euler's load which is partially tensioned is considered. On one of system's ends the discrete element in a form of translational spring which limits the longitudinal displacement was used. The boundary problem of free vibration of the considered system was formulated on the basis of Hamilton's principle and solved according to the small parameter method due to nonlinearities. In the boundary problem formulation process the Bernoulli-Euler theory was used. The relationships between free vibration frequency and parameters such as external load magnitude, translational spring stiffness used for longitudinal displacement control and external load location were studied.

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1. Introduction

Theoretical and numerical investigations of vibration and stability problems of the slender systems (beams [1], columns [2,3,4] and frames [5]) have been discussed by many authors. For non-linear systems the boundary problem is formulated on the basis of: Bernoulli – Euler theory [5,6], non-linear theory at moderately large deflections [3,4], the small parameter method [2] (perturbation method). In the literature one can be found also the investigations on the vibrations of columns [7,8] or beams [9,10], where the theory of beams was proposed by Timoshenko (used when the shear energy and the rotational inertia energy of cross section are contemplated). In publications [11,12] the fixed-fixed column (partially tensioned) subjected to axial Euler's load has been considered without additional elements influences on vibrations or stability. The influence of the elastic boundary conditions on the loss of stability

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was discussed in [6]. The load was placed between the fixed ends of the structure and its point of location was changed along length of the column. The results have shown that the first natural vibration frequency of the investigated structure depends both on the point of location and magnitude of the external force. The non-linear component of free vibrations which depends on amplitude was also taken into account. Influence of an amplitude on natural vibration frequency of geometrically nonlinear multi-member column can be found in the studies [13,14] in which the results for rectilinear [13] and curvilinear [14] forms of static equilibrium were shown.

Additional discrete elements can affect the natural vibration frequency due to reduction of the longitudinal displacement of one of ends [15]. The translational spring limits the axial displacement and can be adapted to the real structure or device.

The main scope of this paper is to present the results of the studies on the relationship between free vibration frequency and parameters such as location of external load, translational spring stiffness (for longitudinal displacement control) of the partially tensioned geometrically non-linear column. Considered system can model a vertical lift with screw-drive system where a nut moves along its length and can be subjected to different types of external load. Translational spring models vibration isolator placed on the upper fixing of mechanical screw. Moreover in this work an influence of the amplitude on free vibrations frequency has been presented.

2. Boundary problem

The considered slender system in the form of a rod subjected to Euler's load is shown in fig. 1. The Euler's load is a compressive external force that does not change the line of action during the system deflection (the line of action is always compatible with the undeformed axis of the column). The Euler's force is placed between the ends of the system. In order to build a mathematical model of the column, it was divided into two segments with lengths l_1 and l_2. The element of length l_1 is compressed while the l_2 is tensioned. Additionally one end of the column is connected to the translational spring with a linear characteristic what affects longitudinal displacement of this end. The investigated system is a non-linear one, so that the vibration frequency dependents on an amplitude of vibration. The size of an influence of an amplitude on free vibration frequency depends on translational spring stiffness.

Boundary problem is formulated on the basis of the Hamilton's principle using non-linear relationship: deformation - displacement and the theory of Bernoulli - Euler. Differential equations of motion in the transversal and longitudinal directions are as follows [11,12]:

$$\frac{\partial^4 w_i(\xi_i,\tau)}{\partial \xi_i^4} + k_i^2(\tau)\frac{\partial^2 w_i(\xi_i,\tau)}{\partial \xi_i^2} + \Omega_i^2\,\frac{\partial^2 w_i(\xi_i,\tau)}{\partial \tau^2} = 0 \tag{1}$$

$$u_i(\xi_i,\tau) - u_i(0,\tau) = -\frac{k_i^2(\tau)}{\theta_i}\xi_i - \frac{1}{2}\int_0^{\xi_i}\left(\frac{\partial w_i(\xi_i)}{\partial \xi_i}\right)^2 d\xi_i \tag{2}$$

Equations (1) and (2) are in the non-dimensional form, where:

$$\xi_i = \frac{x_i}{l_i},\ w_i(\xi_i,\tau) = \frac{W_i(x_i,\tau)}{l_i},\ u_i(\xi_i,\tau) = \frac{U_i(x_i,\tau)}{l_i},\ k_i^2(\tau) = \frac{S_i(\tau) l_i^2}{(EJ)_i}, \tag{3a-d}$$

$$\Omega_i^2 = \frac{(\rho A)_i\,\omega^2 l_i^4}{(EJ)_i},\ \tau = \omega t,\ \theta_i = \frac{A_i l_i^2}{J_i},\ i = 1, 2. \tag{3e-g}$$