Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 153 (2016) 37 – 44

Procedia Engineering

www.elsevier.com/locate/procedia

XXV Polish – Russian – Slovak Seminar “Theoretical Foundation of Civil Engineering”

Way of Optimization of Stress State of Elements of Concrete Structures

Vladimir I. Andreev[a]*, Elena V. Barmenkova[b], Ivan A. Potekhin[c],

[a] *Moscow State University of Civil Engineering (Nacional Recearh University), 26 Yaroslavskoye Shosse, Moscow, 129337, Russia,*
[b] *RUDN University, , Miklukho-Maklaya str. 6, Moscow 117198, Russian Federation*
[c] *Kostroma State Agricultural Academy, Kostroma 156530, Russian Federation*

Abstract

Inverse problems of the theory of elasticity for inhomogeneous bodies, as a rule, are used to identify the dependence of the mechanical properties of the material from the coordinates. It is known that in a thick-walled cylindrical or spherical shell under internal or external pressure the highest stresses are occurred close to the internal surface of the shell. The article presents several solutions of inverse problems, which are defined dependence of the elastic modulus of the radius, at which the equivalent stress in the shell is constant. Corresponding shells can be called equal stress shells. In most cases the change of the module of elasticity of a material leads to changes in its strength. It is shown that for some materials it is possible to create model of equal strength shell in which equivalent stress in each point is equal to material strength. In the article is also considered the way of modification material’s deformation properties at which beams and rods, working under combined stress state (for example, the eccentric compression of columns, the strip foundation and so forth), are also will be equal stress.

Peer-review under responsibility of the organizing committee of the XXV Polish – Russian – Slovak Seminar “Theoretical Foundation of Civil Engineering”.

Keywords: method structural mechanic, inhomogeneity, optimization, iterative method, inverse problem.

1. Introduction

It is difficult to present a construction which at achievement by loadings of limit values would collapse in all points at the same time. From the point of view of solid mechanic, this means that the equivalent stresses, which

* Corresponding author. Tel.: +7-985-222-50-14; fax: +7-499-183-56-83.
E-mail address: asv@mgsu.ru

Peer-review under responsibility of the organizing committee of the XXV Polish – Russian – Slovak Seminar “Theoretical Foundation of Civil Engineering”.
doi:10.1016/j.proeng.2016.08.077

corresponding to the theory of strength for a given material, should be the same at all points of the body. The simplest example of equal stress construction is stretched or compressed rod (see Fig. 1, *a*). In accordance with the hypotheses, adopted in the strength of materials, the normal stresses are constant and equal $\sigma = F / A$ at all points. Without considering the features which arise in the application of load and attachment, according to Saint-Venant's principle, we consider that the rod, given in the example, is equal stress construction. Equal stress construction is not always equal strength. For this purpose it is necessary that not only equivalent stresses in all points of a body was identical, but also strength was also constant. If we imagine a rod which consists of two sections from different materials (see Fig. 1, *b*), it will be equal stress, but won't be equal strength – destruction will happen on that site of a rod which material has smaller strength.

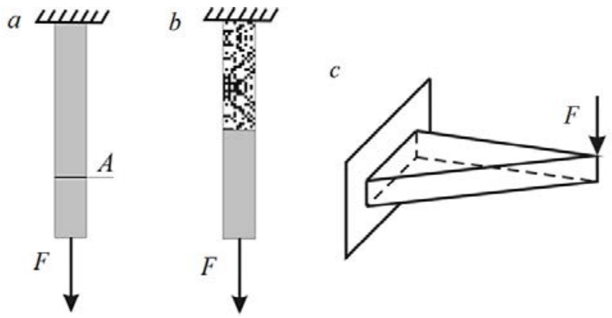

Fig. 1. Tension the rod (*a*, *b*); the beam of equal resistance (*c*).

Other well-known example is the beam of equal resistance at the bend, presented in Fig. 1, *c*. This construction is often called equal strength, although, according to the definition, it is not. The maximum stresses are the same in each section of the beam, i.e. all sections (but not the all beam in general) should ideally reach the limit state at the same time. And, of course, this beam is not equal stress, because stresses change across the sections.

2. Creation the models of equal stress construction

2.1 The thick-walled cylindrical shell

On the example of a task of Lame's problem for a thick-walled shell (Fig. 2, *a*) we will consider the way of creation the models of equal stress construction on the basis of the solution of the return task of inverse problems of the theory of elasticity of inhomogeneous bodies. We will explain the main idea of a method. In Fig. 2, *b* by a dashed line are shown the diagrams of stresses in the loaded by internal pressure pipe, which material is homogeneous, i.e. the elasticity module $E = E_0 = const$ (a dashed line in Fig. 2, *c*). If for simplicity to accept that for this material the theory of strength of the maximum normal stresses is fair, then at achievement by pressure of limit value destruction will begin on an internal contour of a pipe, where $\sigma_\theta = \sigma_{max}$. So, the remainder of the pipe is underused.

From the theory of elasticity of inhomogeneous bodies [1] it is known that in inhomogeneous bodies in zones where the module of elasticity is less, stresses decreases in comparison with homogeneous material, and vice versa. Fig. 2, *b*, by the solid line show the qualitative diagrams corresponding to a solution for the pipe, in which the modulus of elasticity is a function $E = E(r)$ (see Fig. 2, *c*). Note that the influence of inhomogeneity has little effect on the stresses σ_r, and the diagram σ_θ is changed significantly, approaching to the constant.

In [2 - 5] solved some problems in finding such dependence $E(r)$, at which the equivalent stresses corresponding to the various theories of strength will be constant. These are so-called inverse problems in which we seek a functions of changing of the modulus of elasticity for a given stress state of the structure. So, for example, if for a material the theory of strength of the maximum shear stresses is true, under the condition that in a pipe $\tau_{max} = (\sigma_\theta - \sigma_r) / 2 = const$, this function has the form: