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# Application of the Contact Layer in the Solution of the Problem of Bending the Multilayer Beam

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**Abstract**

The article deals with solution for stress-strain state of multilayer composite beams of rectangular cross-section, which is bended by the normally distributed load . The interaction between layers is accomplished by the contact layer, in which the substances of adhesive and substrata are mixed.

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*Keywords:* bending, composite, multilayer beam, contact layer, edge effect

**Nomenclature**

| | |
|---|---|
| $k$ | layer number |
| $\varepsilon_{f,k}$ | forced deformations |
| $B_k$, $C_k$, $D_k$ | averaged characteristics of the cross-section |
| $N_{f,k}$, $M_{f,k}$ | forced internal forces |
| $\tau^*_{yx,k}$, $\tau^*_{yz,k}$, $\sigma^*_{y,k}$ | stresses in contact layer |
| $gr^*_k$, $er^*_k$ | averaged characteristics of the contact layer |

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## 1. Introduction

Using the contact layer in the solution of the problem allows us to overcome problems such as endless tangential stresses arising at the border between layers and allows to calculate physical properties of the contact layer on the basis of experimental data [1].

We will consider the contact layer as the transversal anisotropic medium with such parameters that it can be represented as a set of short elastic rods, which are not connected to each other. For simplicity, we assume that the rods are normally oriented to the contact surface.

## 2. Statement of the Problem

Fig. 1 shows a cross-section of beam with any two layers ($k-1$ and $k$). Rigid connection between layers is accomplished by the contact layer, in which the substances of two layers are mixed.

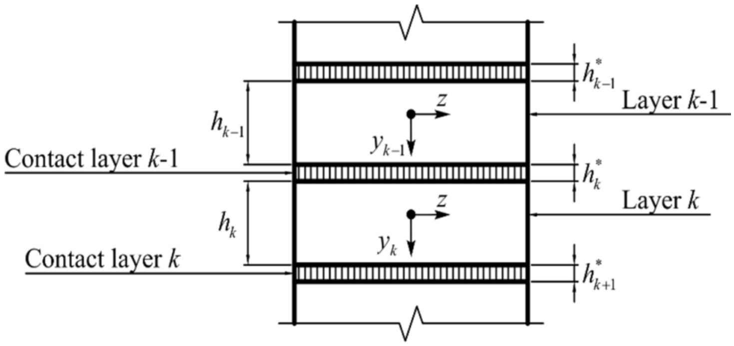


Fig. 1. The model of a beam's cross-section

Here and further, all values marked with * relate to a contact layer.

## 3. Derivation of Resolving Equations

In accordance with the hypothesis of beams, displacement and strain of any $k$-th layer can be written as,

$$u_k = u_{0,k} - y_k \cdot \frac{\partial \upsilon_k}{\partial x}; \;\; \varepsilon_{x,k} = \frac{\partial u_k}{\partial x};$$
$$\varepsilon_{x,k} = \varepsilon_{0x,k} + \frac{y_k}{\rho_k} = \varepsilon_{0,k} + R_k \cdot y_k, \quad (1)$$

where $u_{0,k}$ is the displacement at the neutral axis; $\varepsilon_{0x,k}$ is the strain at the neutral axis; $\rho_k$ is the radius of curvature; $R_k$ is the curvature of the cross-section.

For beams Hooke's law takes the following form:

$$\varepsilon_{x,k} = \frac{1}{E_k} \cdot \left[\sigma_{x,k} - \nu_k \cdot \left(\sigma_{y,k} + \sigma_{z,k}\right)\right] + \varepsilon_{f,k} \approx \frac{\sigma_{x,k}}{E_k} + \varepsilon_{f,k}, \quad (2)$$

from which it follows

$$\sigma_{x,k} = E_k \cdot \left(\varepsilon_{x,k} - \varepsilon_{f,k}\right), \quad (3)$$

where $\varepsilon_{f,k}$ are forced deformations.