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Verification of tensegrity properties of Kono structure and Blur building

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Abstract

Cable-bar structures are commonly used in engineering. Some of them are in special configuration named "tensegrity". The objective of the present paper is to describe and verify two interesting examples of existing civil engineering structures called tensegrity in the literature: Kono structure and Blur building. Singular value decomposition of the compatibility matrix of truss structures is used to define two crucial features of tensegrity: existence of mechanisms and self-stress states. Eigensolution of the stiffness matrix extended on geometrical stiffness matrix allow to identify if the self -stress stabilizes the equilibrium of the structure – if the mechanisms are infinitesimal or finite.

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1. Introduction

Nowadays the concept of tensegrity structures [4,6] is understood in many ways [12]. A widely accepted version is proposed by Pugh [13] as follows: "A tensegrity system is established when a set of discontinuous compressive components interacts with a set of continuous tensile components to define a stable volume in space."

Key features of tensegrity structures can be defined as [8]:

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– the structure is a truss,
– there are self-stress sates,
– there are infinitesimal mechanisms and they are stabilized by these self-stress states,
– the system of compression elements is discontinuous,
– compression members are inside the tensile elements,
– tensile elements have no rigidity – they are strings.

The presence of all of these characteristics allows qualifying a structure as a tensegrity structure.

A key step in the design of tensegrity structures is the determination of their geometrical configuration, known as form-finding. The form-finding process determines a possible pre-stress distribution and geometry for a tensegrity.

The method based on the singular value decomposition (SVD) [5,9] is used in the paper to identify whether the structure is geometrically variable and whether there are self-stress states.

The term "tensegrity" is so attractive that many engineering structures are called with its use. Two interesting examples of existing civil engineering structures are presented and analyzed in Chapter 3.

2. SVD decomposition

The subject of the analysis is N-membered, supported truss with following characteristics: material constants E_e, cross-sectional areas A_e and bar lengths L_e. Its mechanical properties are described by three linearized equations: compatibility, material properties and equilibrium with boundary conditions included

$$\mathbf{\Delta} = \mathbf{B}\mathbf{q}\,,\ \ \mathbf{S} = \mathbf{E}\mathbf{\Delta}\,, \mathbf{B}^T\mathbf{S} = \mathbf{P} \tag{1}$$

where **q** is displacement vector of length M, **Δ** is extension vector, **S** is normal force vector, **E** is elasticity matrix, **P** is load vector and **B** is compatibility matrix which can be determined directly or using the formalism of the finite element method [8,14]. The singular value decomposition of an N×M real matrix **B** is a factorization of the form:

$$\mathbf{B} = \mathbf{Y}\mathbf{N}\mathbf{X}^{\,T} \tag{2}$$

where **Y** is an N×N real orthogonal matrix, **X** is an M×M real orthogonal matrix and **N** is an N×M rectangular diagonal matrix. Let us consider two eigen problems

$$\left(\mathbf{B}\mathbf{B}^T - \mu\,\mathbf{I}\right)\mathbf{y} = 0 \ \text{ and } \left(\mathbf{B}^T\mathbf{B} - \lambda\,\mathbf{I}\right)\mathbf{x} = 0 \tag{3}$$

with the solutions in the form of eigenvalues and eigenvectors (normalized)

$$\mu_1, \mathbf{y}_1;\ \mu_2, \mathbf{y}_2;\ ...;\ \mu_N, \mathbf{y}_N \ \text{ and } \lambda_1, \mathbf{x}_1;\ \lambda_2, \mathbf{x}_2;\ ...;\ \lambda_N, \mathbf{x}_N \tag{4}$$

Full solutions of the above eigen-problems can be expressesd in the condensed forms

$$\mathbf{B}\mathbf{B}^T = \mathbf{Y}\mathbf{M}\mathbf{Y}^T \ \text{ and } \mathbf{B}^T\mathbf{B} = \mathbf{X}\mathbf{L}\mathbf{X}^T \tag{5}$$

$$\mathbf{M} = diag\{\mu_1 \quad \mu_1 \quad ... \quad \mu_N\},\ \mathbf{L} = diag\{\lambda_1 \quad \lambda_1 \quad ... \quad \lambda_M\},\ \ \mathbf{Y} = [\mathbf{y}_1\ \mathbf{y}_2\ ...\ \mathbf{y}_N],\ \ \mathbf{X} = [\mathbf{x}_1\ \mathbf{x}_2\ ...\mathbf{x}_M] \tag{6}$$

One can notice that the product $\mathbf{BB^T}$ can be considered as a matrix of symmetrised equations of equilibrium with non-negative eigenvalues. Zero eigenvalues (if any) are related to the non-zero solution of homogeneaous equations (**P**=**0)** named self-stress. The self-stress can be considered as an eigenvector related to zero eigenvalue).

In a similar way the product $\mathbf{B^TB}$ can be considered as a particular form of linear stiffness matrix with unit elasticity matrix. The eigenvalues are non-negative. Zero eigenvalues (if any) are related to the finite or infinitesimal mechanisms, but in general the information from the null-space analysis alone does not suffice to establish the difference. The mechanism can be considered as an eigenvector related to zero eigenvalue.

Based on the above two eigen-problems it is easy to proof the singular value decomposition of the matrix **B**

$$\mathbf{B}\mathbf{B}^T = \mathbf{Y}\mathbf{N}\mathbf{X}^T\mathbf{X}\mathbf{N}^T\mathbf{Y}^T = \mathbf{Y}\mathbf{N}\mathbf{N}^{\,T}\mathbf{Y}^T = \mathbf{Y}\mathbf{M}\mathbf{Y}^{\,T} \ \text{ and } \mathbf{B}^T\mathbf{B} = \mathbf{X}\mathbf{N}^{\,T}\mathbf{Y}^T\mathbf{Y}\mathbf{N}\mathbf{X}^T = \mathbf{X}\mathbf{N}^T\mathbf{N}\mathbf{X}^T = \mathbf{X}\mathbf{L}\mathbf{X}^T \tag{7}$$