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Dissection Method Applications for Complex Shaped Membranes and Plates

R.F. Gabbasov[a],V.V. Filotov[a],N.B. Ovarova[a], A.M. Mansour[a]*

[a]*Moscow State University of Civil Engineering (National Research University), 26 Yaroslavskoye Shosse, Moscow, 129337, Russia*

Abstract

Dissection (disintegration) for Poisson’s two-dimensional problems differential equation takes us to 2nd-order ordinary differential equations, the thing that simplifies a solution algorithm and makes it programmable. This algorithm illustrated by examples of membranes and calculations of bend plates. New obtained results are compared with those known and stated in the references listed at the last part of the article. The new algorithm obtained by dissecting ordinary differential equations of the 1st-order, based on applying the method of successive approximations and the generalized equations of finite difference method.
Solution convergence are illustrated in both methods, also; the out coming algorithm shows the great combination between both techniques. The super advantage of the new developed algorithm is the capability of calculating plates sophistically shaped, which stands as a great benefit for engineers and designers.

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Keywords: Plates; membranes; differential equations; numerical solutions; successive approximation; finite difference

Nomenclature

w	Dimensionless Deflection
ξ , η	Cartesian Coordinates
p	Distributed Load
h	Mesh Spacing (Step)
i	Measuring Along The Axis (ξ)
j	Measuring Along The Axis (η)

* Corresponding author. Tel.: +7-985-366-3123.
E-mail address: alaa_for_all@hotmail.com

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n	Number of Domain Divisions (Meshes)
v	Poisson's Coefficient
a	Plate Side Length
q_0	Load Intensity at a Certain Node

1. Main text

In engineering practice, plates of different shapes are widely used. The far known numerical methods and techniques allow engineers to calculate such complicated plates, but they are quite complex.

This article presents the dissection method (*disintegration*), and numerical implementation represented by the combination of successive approximations method and generalized equations of finite difference method. It also targets a development of highly accurate and simple algorithm for calculating plates and membranes in complex shapes.

Considering dissection method to calculate a strained membrane, the following differential equation in partial derivatives describes the strain-load relation:

$$\frac{\partial^2 w}{\partial \xi^2} + \frac{\partial^2 w}{\partial \eta^2} = -p(\xi, \eta) \tag{1}$$

It's well known that the dissection method is applicable only for differential equations systems, in which the differential operator is as the sum of one-dimensional differential operators. (***Eq.1***) satisfies this condition. Suppose

$$\frac{\partial^2 w}{\partial \xi^2} = -w^{\xi\xi} \tag{2}$$

Thus,

$$\frac{\partial^2 w}{\partial \eta^2} = -p + w^{\xi\xi}. \tag{3}$$

Consequently, instead of (***Eq.1***); we have a system of two homogeneous algebraic equations, equations for such difference operators are formulated much easier. Now, the difference equations (*successive approximations*), approximating (***Eq.2***) and (***Eq.3***) considering uniform mesh with no discontinuities, this produces:

$$w_{i-1,j} - 2w_{i,j} + w_{i+1,j} = -\frac{h^2}{12}\left(w^{\xi\xi}_{i-1,j} + 10w^{\xi\xi}_{i,j} + w^{\xi\xi}_{i+1,j}\right) \tag{4}$$

$$w_{i,j-1} - 2w_{i,j} + w_{i,j+1} = -\frac{h^2}{12}\left(p_{i,j-1} + 10\, p_{i,j} + p_{i,j+1} - w^{\xi\xi}_{i,j-1} - 10w^{\xi\xi}_{i,j} - w^{\xi\xi}_{i,j+1}\right) \tag{5}$$

Then, problem is solved using Seidel's iterative method, initially conditioned to zero, that results in converting (***Eq.4***) and (***Eq.5***) to the form:

$$w^{\xi\xi}_{i,j} = -\frac{1}{4}(v_{i-1,j} - 2v_{i,j} + v_{i+1,j}) - \frac{1}{24}(w^{\xi\xi}_{i-1,j} - 14w^{\xi\xi}_{i,j} + w^{\xi\xi}_{i+1,j}) \tag{6}$$

$$v_{i,j} = \frac{1}{2}(v_{i,j-1} + v_{i,j+1}) + \frac{1}{12}(p_{i,j-1} + 10p_{i,j} + p_{i,j+1} - w^{\xi\xi}_{i,j-1} - 10w^{\xi\xi}_{i,j} - w^{\xi}_{i,j+11} \tag{7}$$