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About method of basis (local) variations and its applications in structural analysis

Marina L. Mozgaleva[a], Pavel A. Akimov[a,b,c]*, Zhiuli I. Mskhalaya[a], Vladimir A. Kharitonov[a]

[a]*Moscow State University of Civil Engineering (National Research University), 26 Yaroslavskoye Chaussee, Moscow, 129337, Russia*
[b]*Russian Academy of Architecture and Construction Sciences, 24, Ulitsa Bolshaya Dmitrovka, Moscow, 107031, Russia*
[c]*Research & Educational Center “StaDyO”, Office 810, 18, 3-ya Ulitsa Yamskogo Polya, Moscow, 125040, Russia*

Abstract

This paper is devoted to so-called method of basis (local) variations, proposed by A.B. Zolotov, and its applications in structural analysis. Combination of method of basis variation and effectively with finite element method (FEM) or variation-difference method (VDM) for formulation of resolving (resultant) system of linear algebraic equations. Theoretical foundations of this method are presented. Basic formulas for linear and non-linear problems of structural mechanics are given.

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1. Introduction

As known majority of problems of structural mechanics [1,2], one way or another, can be formulated with the use of the concepts of the theory of linear operators and linear spaces [3]. We should mention here, in particular, concepts of vector, linear operator, linear and bilinear functionals. Equations of general theory of elasticity [2] and equations of plate analysis [4,5] are vital samples of linear differential operators. Expressions of energy conservation law or work of external forces (applied to structure) on displacements [1,2] are samples of bilinear and quadratic functionals. Besides,

* Corresponding author. Tel.: +7(495)625-71-63; fax: +7(495)650-27-31.
E-mail address: akimov@raasn.ru

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equation for work of external forces (applied to structure) is linear functional from a mathematical standpoint. It can be shown after analysis of formulations of boundary problems, that these formulations contain common part (description of the given domain and a variety of considering functions within solving process) and different mathematical equations for the conditions over the given domain and conditions at the boundary of domain. We have a problem similar to the computing of certain integrals of the following form

$$J = \int_{\Omega} F(x)dx$$

with the given domain and function. If we consider the integral over extended domain (within method of extended domain [6]), the problem reduces to the computing of the integral of the form

$$J = \int_{\omega} \theta(x)F(x)dx\,,$$

where $\theta(x)$ is characteristic function of domain Ω ; $x = (x_1, x_2, ..., x_N)$. Function $\theta(x)$, domain discretization and the given function $F(x)$ are normally used as formal parameters for developing software in such cases.

Conversely universal and the most convenient (variational) formulation of the boundary problem has the form

$$\min \Phi(u) = \int_{\omega} \theta(x)F(x,u)dx\,. \tag{1}$$

Expression (1) describes conditions over the given domain (differential equations) and natural boundary conditions. In the case of the given principal boundary conditions, solution of the problem is searched at least on the set of functions satisfying these conditions. In this sense, the development of corresponding software is required for solving of boundary problems with functions $\theta(x)$ and $F(x,u)$ as formal parameters. Variational formulations corresponds to a wide class of physical models (in addition to the common specification), have a smaller (two times less) order of derivatives of the function u and includes the most difficult in terms of approximation boundary conditions (natural boundary conditions). Compatibility of conditions over the given domain, boundary conditions and right-side function is important factor.

Application of standard approximation mesh simplifies approximation problems. It is carried out by introducing a mesh consisting in particular of the convex quadrilateral (two-dimensional) or octagonal (three-dimensional) elements. Since the mesh is normally defined as topologically equivalent to rectangular with a single mesh width, the new local coordinate system is introduced and corresponding mesh converts into a rectangular mesh with a single mesh width, corresponding integrals are computed over elements in the local coordinates. Thus, initial data for software development includes functions $\theta(x)$, $F(x,u)$ and coordinates of mesh nodes (it should be noted that only the second factor depends on differential equations, describing conditions over domain). Formally, approximation of the boundary problem confined to definition of approximation mesh, shape functions and integration scheme (normally trapezoid formula) within mesh element (for instance, finite element [2]). Further algorithm for constructing of resultant systems of linear algebraic equations and their solution by direct methods (for instance, Gauss method) for linear problems, and by iterative methods shouldn't depend on the type of boundary problem at all.

2. Theoretical foundations of method of basis (local) variations

2.1. Linear finite-dimensional operator

As known operator L is transformation $L: V \to W$, which put element $\bar{y}$ from space W in correspondence with element $\bar{x}$ from space V ($\bar{y} = L\bar{x}$). Besides, we have linear operator L if the following formulas (additivity (homogeneity) property) are valid for arbitrary elements $\bar{x}_1$, $\bar{x}_2$ from space V and arbitrary complex number λ