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MATHEMATICAL MODELING OF THE STIRLING ENGINE

Wrona Jan[a], Prymon Marek[b] *

[a]*Cracow University of Technology, Faculty of Environmental Engineering, ul. Warszawska 24, Cracow 31-155, Poland*
[b]*Thessla Green sp. z o.o., ul. Makuszyńskiego 4a , Cracow 31-752, Poland*

Abstract

The paper presents mathematical models which have been developed by the authors, and the results of which may be used to design an experimental refrigeration unit operating in the Stirling cycle. The systems engineered and based on Stirling cycle may be considered as an alternative to commonly employed internal combustion engines and Linde circulation cooling systems. The article presents a time discretization model assuming the cylinders as adiabatic spaces. The model enables the size optimization of all particular elements of the Stirling device such as: heat exchangers, the regenerator, the cylinders, piston motion and phase displacement. The advantage of modelling is the calculation speed when compared to modelling based on full Navier-Stokes system of equations (CFD) therefore enabling the dimensioning of the device. The results obtained using the simplified model have been verified by the other modelling type – that is the full 3D CFD, in which the whole working space including the heat exchangers and the regenerator has been modelled. Additionally, a dynamic MESH option has been applied in order to simulate the movement of the pistons in the cylinders.

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1. Introduction

The Stirling cycle was invented by a Scottish clergyman, Robert Stirling. He obtained a patent for the invention

* Corresponding author. Tel.048-12-628-2819.
E-mail address: jwrona@pk.edu.pl

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at the beginning of the 19th century in 1816 and two years later the first working version was used to pump out water from a quarry. Despite its theoretical efficiency being equal to the efficiency of the Carnot cycle, the development of Stirling engine was not as dynamic as the evolution and expansion of the steam engine or internal combustion engine. The main obstacle in the design was the shortcomings in materials necessary to build the working unit and a very complex thermodynamic description (difficult to depict at that time). The advancement in material technology especially in the second half of the 20th century and the development in computer technology employing numerical calculation methods are the reasons why technological barriers from more than 100 years ago are now history.

Engineers and scientists are presently rediscovering the concept of the Stirling engine because it possesses many positive attributes such as high theoretical efficiency, compact and relatively simple design, environmentally inoffensive working medium. All of these make it ideal for contemporary conditions. An almost forgotten 200 year-old idea has revived in our times and has the potential to stand up to currently used engines employing thermodynamic cycle in wide variety of field technology from cooling systems, industrial engines and cogeneration [3, 4, 7, 10, 14, 20].

Devices employing the Stirling cycle are still expensive to build simply because of limited production orders. In case of Stirling engine specific working conditions (high temperature and high pressure) have a detrimental effect on the price, especially the cost of the prototype unit. To offset the high cost, the designers try to lower it by implementing new materials and search for optimum work parameters lowering the average pressure in the working chamber. In case of cooling devices where there is no need for high work temperature the mass production cost can be reduced significantly. Initiating production of coolers of any kind employing the Stirling cycle is justified as the cooling medium widely used in contemporary cooling systems has a devastating effect on the natural environment contributing to the ozone layer depletion and the greenhouse effect.

Nomenclature

M	total gas mass in the machine, kg
m_C	mass of gas in a warm cylinder, kg
m_E	mass of gas in a cold cylinder, kg
m_{HC}	mass of gas in a warm exchanger, kg
m_{HE}	mass of gas in a cold exchanger, kg
m_R -	mass of gas in a regenerator exchanger, kg
T_C	temperature in the warm cylinder, ^oC
T_{HC}	temperature in the heat exchanger, ^oC
T_R	temperature in the regenerator, ^oC
T_{HE}	temperature in the cold exchanger, ^oC
T_E	temperature in the cold cylinder, ^oC
c_p	specific heat at constant pressure, J/kgK
c_v	specific heat at constant volume, J/kgK
$V_C = V_C(\emptyset)$	volume of hot cylinder, m^3
$V_E = V_E(\emptyset)$	volume of cold cylinder, m^3
$V_{HC} = const$	volume of hot exchanger, m^3
$V_{HE} = const$	volume of cold exchanger, m^3
$V_R = const$	volume of regenerator, m^3
W	work of thermal cycle, W
Q	total amount of the heat (Fig. 2), W
p	pressure, Pa
Φ	actual instantaneous shaft angle position
γ	coefficient of compressibility