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A Study on uncertainty of discharge in river channel using stochastic partial differential equation

Daiwei Cheng[a]* , Yamada Tadashi[b]

[a]*Graduate School of Science and Engineering, Chuo University, 1-13-27, Kasuga, Bunkyo- ku, Tokyo, 112-8851, Japan*
[b]*Faculty of Science and Engineering, Chuo University, 1-13-27, Kasuga, Bunkyo-ku, Tokyo, 112-8851, Japan*

Abstract

The physical systems are modeled by differential equations in engineering problems. If all the parameters involved in a physical system can be known, with the initial and boundary conditions, one can predict the time evolution of the physical system by solving the govern differential equations.

But the more general case is that we do not know the parameters and conditions precisely because the real problems in engineering are all governed by some very complex nonlinear systems. So that the parameters and initial, boundary conditions may fluctuate randomly or at least it appears to us in that way.

One way to deal with that situation is to use stochastic differential equations. The theory of stochastic ordinary differential equations has been well developed since K. Ito [2] introduced the stochastic integral and the stochastic integral equation in the mid-1940s. Recently, K. Yoshimi and T. Yamada [5] have used this method to study the uncertainty of discharge due to the random fluctuation in precipitation.

The present study tried to general Ito's method to the shallow water equation which is a partial differential equation, to study the uncertainty of discharge due to the randomness effects in space by changing the partial differential equation to an ordinary differential equations using the Lagrange point of view. As a result, we proposed a Boltzmann-type equation to govern the time evolution of the probability density function of discharge and water level in river channel and show the result of simulation.

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* Corresponding author.
E-mail address: davidchengchina@gmail.com

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1. Introduction

Generally speaking, researchers are preferring deterministic method to stochastic method in the hydrology calculations, such as rainfall-runoff simulation, or open channel simulation. The reason is that the computational cost of stochastic method is much higher than deterministic methods. However, in atmosphere science or ocean engineering, researchers are more likely to use stochastic method or ensemble forecast even though the computational cost is already very expensive in these fields.

The problem of deterministic method was pointed out by Edward Lorenz in 1963. It is now well known as the “chaos phenomenon”. Chaos phenomenon can be described as follows: A phenomenon that the time evolution of a dynamical system is highly sensitive to the initial condition, usually because of the strong nonlinearity of the system. Because it is almost impossible to determine the exact initial condition of a complex system in atmosphere science, a technique named “data assimilation” which combined stochastic method and ensemble forecast is rapidly developing in recent years.

It is true that the nonlinearity of hydrology systems may not as strong as atmosphere systems, but it becomes more and more important to understand the uncertainty in hydrology system from the point of view of disaster risk management. So the present study is aimed to suggest a method to consider the spatial uncertainty of open channel when doing unsteady flow simulation.

2. Basic of stochastic differential equation and its relation to data assimilation

In this section we will discuss the basic of stochastic differential equations. And show how stochastic differential equations theory play an important part in data assimilation technique.

2.1. Basic of stochastic differential equations

The history of stochastic differential equations can be traced back to Albert Einstein’s paper in 1900 [1] which discusses Brownian motion. Later K. Ito [2] introduced the stochastic integral and the stochastic integral equation in the mid-1940s. He also showed that the time evolution of the random variables’ probability density function in stochastic differential equations follows the Fokker-Planck equation.

stochastic differential equations are usually written in the following form:

$$d\vec{X} = \vec{F}(\vec{X},t)dt + \vec{\vec{\sigma}}(\vec{X},t)d\overrightarrow{W} \tag{1}$$

This form was suggested by K. Ito. $\vec{X} = (x_1, x_2, \cdots x_n)$is a n-dimensional vector which represents the state of the system, $\vec{F}(\vec{X},t)$is the deterministic part of the system $\vec{\sigma}(\vec{X},t)$is the covariance matrix of the random external force, and $d\overrightarrow{W}$ is a standard multivariate Winner process. The equation described the time development of a system under random external force.

K. Ito [2] have also shown that the equation (1) is equivalent to the following n-dimensional Fokker-Planck equation.

$$\begin{aligned} \frac{\partial p(\vec{X},t)}{\partial t} &= -\sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left(f_i(\vec{X},t)p(\vec{X},t)\right) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial^2}{\partial x_i \partial x_j}\left(D_{ij}(\vec{X},t)p(\vec{X},t)\right) \\ D_{ij}(\vec{X},t) &= \sum_{k=1}^{n} \sigma_{ik}(\vec{X},t)\sigma_{kj}(\vec{X},t) = \vec{\vec{\sigma}}(\vec{X},t)\vec{\vec{\sigma}}^T(\vec{X},t) \end{aligned} \tag{2}$$