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Fundamentals of Continuous Growth Processes in Technology and Nature

Alexander V. Manzhirov[a,*], Narinder K. Gupta[b]

[a] *Institute for Problems in Mechanics of the Russian Academy of Sciences,Vernadsky Ave 101 Bldg 1, Moscow, 119526, Russia; Bauman Moscow State Technical University, 2nd Baumanskaya Str 5/1, Moscow, 105005, Russia; National Research Nuclear University TMEPhIT, Kashirskoye shosse 31, Moscow, 115409, Russia; Moscow Technological University, Vernadsky Ave 78, Moscow, 119454, Russia*

[b]*Indian Institute of Technology Delhi, Hauz Khas, New Delhi, 110016, India*

Abstract

Fundamentals of mathematical modeling of surface growth process are under consideration. Mathematical models of 3D and 2D solids that grow owing to processes of continuous influx of new elements (surfaces and fibers) are proposed. Complete systems of equations and boundary value problems are obtained. A method for solving these boundary value problems is developed. Qualitative conclusions and numerical results concerning the behavior of growing solids are presented.

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1. Introduction

There are two main ways of continuous growth in technology and nature known to researchers. These are volumetric and surface growth processes. Consider the latter case with a reference to the monographs [1, 2], where fundamental results concerning volumetric growth have been presented.

A vast majority of objects or solids that surround us arise from some surface growth processes. As an example, one can present many technologies in industry, including well-known technologies of crystal growth, laser deposition, solidification of melts, electrolytic formation, pyrolytic deposition, polymerization, concreting, and modern digital additive manufacturing technologies. Similar processes determine the specific features of natural phenomena such as the growth of biological tissues, glaciers, blocks of sedimentary and volcanic rocks, space objects, etc. Recent research has shown that solids formed by growth processes differ in their properties essentially from solids in the traditional view. Moreover, the classical approaches of solid mechanics fail when modeling the growing solid behavior. They should be replaced by new ideas and methods of modern mechanics, mathematics, physics, and engineering sciences.

The approach proposed here deals with the construction of an adequate model of surface growth processes of solids (see also [3–9]). This approach is based on the following statements:

* Alexander V Manzhirov. Tel.: +7-495-434-4138 ; fax: +7-499-739-9531.
E-mail address: manzh@ipmnet.ru

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- We simulate the surface growth of a solid by the motion of its boundary owing to the influx of new material to the surface of this solid.
- We obtain specific boundary conditions on the moving boundary (growth surface) as the result of an additional contact interaction problem between 3D solid and 2D surface, which depends on particular features of the growth process.
- We state the compatibility of the strain rate (or stretch rate tensor) for a growing solid, while its strain tensor is incompatible as a rule.

The last statement leads to the case in which it is absolutely natural to choose the stress rate tensor, the strain rate tensor, and the velocity vector as the basic variables of the governing system of equations for the description of the surface growth process. In general, a boundary value problem for a growing solid contains three dependent controlled groups of values: the surface and bulk loadings, the tension of new adhering surfaces, and the velocity of influx of adhering surfaces.

Deformation processes in a solid whose composition, mass, or volume varies in a piecewise continuous manner owing to the influx of new material are of great interest for engineers, researchers and technologists in numerous areas. The solid mechanics problems arising in the field of modeling of such processes are completely new and form a separate field of research known as mechanics of growing solids. Its importance is determined by the fact that almost all solid objects in nature and technology (buildings, structures, structural components, machine parts, trees, bones, soft tissues, etc.) appear under some growth process. The process of accretion or deposition of new material to a solid is studied in the fundamental scientific area called Mechanics of Growing Solids. This area deals with all sorts of solid materials including elastic, viscoelastic, plastic, composite and graded materials (see, e.g., [1–6, 13]).

Additive manufacturing technologies are a particular case of growth processes. Mathematical modeling of additive manufacturing technologies is aimed at improving the performance of device, machine, and mechanism parts. These technologies include stereolithography, electrolytic deposition, thermal and laser-based 3D printing, 3D-IC fabrication technologies, etc. They are booming nowadays, because they can provide rapid low-cost high-accuracy production of 3D items of arbitrarily complex shape (in theory, from any material). However, deformation and strength problems for products manufactured with these technologies yet remain to be solved. The fundamentally new mathematical models considered in the paper describe the evolution of the end product stress-strain state in additive manufacturing and are of general interest for modern technologies in engineering, medicine, electronics industry, aerospace industry, and other fields (e.g., see [7–12, 14]).

2. Characteristic Features of Growing Solids

By a (piecewise) continuously growing solid we mean a solid whose composition, mass, or volume varies as a result of (piecewise) continuous addition of material to its surface. The process of adding new material to the solid is called accretion, or growth. For piecewise continuous accretion, the following basic stages of its deformation are strictly followed: before accretion, during the continuous growth, and after the accretion has ceased and growth has stopped. Each of these stages is characterized by the times when it starts and ends. The first is characterized by the time of application of a load to the solid and the time when growth starts. The second, by the time when growth starts and the time when it ends. The third is characterized by the time when growth ends and the time when it starts. The process under investigation is usually concluded by the third stage, for which the time when the next stage begins is taken to be as long as desired. The solid on whose surface new material is deposited starting from the time when accretion starts is called the basic or original solid. The solid consisting of the material pieces added to the basic solid over the time interval from the beginning of accretion up to a given instant of time is called the additional solid. The additional solid can have a complex structure and consist of a collection of solids formed over different time intervals of continuous accretion. We call them sub-solids. The additional solid is obviously the union of sub-solids. The domains occupied by the former and latter can be disconnected. The union of the basic and the additional solids will be called the accreted or growing solid. Note that accretion can also occur without the basic solid, starting from an infinitesimal material element. The part of the surface where infinitesimal pieces of the material are deposited at the actual instant is called the accretion or growth surface. The growth surface may be disconnected, in general. In particular, it can be the whole surface of the solid. Finally, the part of the surface of the original or the growing solid