Available online at www.sciencedirect.com
ScienceDirect
Procedia IUTAM 20 (2017) 89 – 93
Procedia IUTAM
www.elsevier.com/locate/procedia

24th International Congress of Theoretical and Applied Mechanics

Why we need more Degrees of Freedom

Shai Revzen[a,*], Daniel E. Koditschek[b]

[a]Electrical Engineering and Computer Science & Ecology and Evolutionary Biology, University of Michigan, Ann Arbor, MI, USA
[b]Electrical and Computer Engineering, University of Pennsylvania, Philadelphia, PA, USA

Abstract

Mechanical systems encountered in biology typically have many more degrees of freedom (DOF) than the 6 DOF required to manipulate a body in space. Even the relatively rigid arthropods and crustaceans have at least 5 DOF in each limb; tentacles and human hands have many more. Robotics engineers are routinely required to choose the number of DOF in a robot in the early design stages, potentially limiting the robot's future uses. We theoretically motivate the definition of "mechanical versatility" as the ability of a mechanical system to express distinct static configurations and switch among them rapidly. Requiring versatility and assuming that the systems are power and force limited, and must furthermore resist finite energy environmental disturbances to their state, we show that such multiuse[1] mechanical systems have a lower bound on the number of DOF they require. For bio-mechanics, this suggests which organs and organisms will be driven to become more complex mechanically by indicating domains where higher DOF systems would intrinsically out-compete lower DOF systems.

Peer-review under responsibility of organizing committee of the 24th International Congress of Theoretical and Applied Mechanics

Keywords: versatility, biomechanics, robotics

Nomenclature

$\mathtt{Q}$	Configuration space
Σ	Alphabet of mechanical "symbols"; a finite discrete subset of $\mathtt{Q}$
q	A configuration, $q \in \mathtt{Q}$
$\mathcal{V}$	The "mechanical versatility" of the system (defined herein)
V_{max}	Maximal velocity through configuration space
F_{max}	Maximal force exerted
W_{max}	Maximal power available
ΔE	Maximal disturbance energy

1

E-mail address: shrevzen@umich.edu

Peer-review under responsibility of organizing committee of the 24th International Congress of Theoretical and Applied Mechanics
doi:10.1016/j.piutam.2017.03.012

1. Background

One of the mysteries of the natural world is the staggering mechanical complexity of organisms[2]. Why does nature select for mechanisms so much more complex than those we build? For example, why did natural selection provide many more DOF in the limbs of an animal than the 6 DOF required to arbitrarily place and orient that limb in space?

In this short paper we offer a motivating argument for why a high DOF count is an inevitable consequence of requiring "mechanical versatility" in a physically limited system. This argument has close ties to the broader question of understanding the limits and benefits of "morphological computation"[3] – the computational contribution of animal (and robot) bodies to their motions. We believe this is among the first results of this kind to be published, and hope it will motivate further research into the fundamental trade-offs inherent in embodiment of agents. The science of both biological and artificial agents could be advanced by understanding the requirements and limitations of embodied cognition[4].

We begin in section 1.1 by motivating and defining a notion of mechanical versatility using notions derived from the theory of computation.

1.1. Mechanical Versatility – a definition

To define "mechanical versatility" we rely on notions from formal language theory in computer science[5]. The intuitive essence of versatility is the presence of multiple capabilities – defined formally by the term "multi-ability" in Ferguson et al.[1] – and the further ability to rapidly switch among those capabilities. Let us indicate each such capability as symbol from a finite alphabet Σ. Versatility is thus the ability to express any string in the language Σ^* generated by this set of symbols Σ.

To obtain a simple and tractable theory of "mechanical versatility" we will take each symbol $\sigma \in \Sigma$ to be some nominal static configuration of the system. By "expressing a symbol σ" we will mean the system needs to maintain its configuration close enough to this nominal static configuration σ. Here we abuse notation, taking $\Sigma \subseteq \mathtt{Q}$ to be a discrete finite subset of the configuration space $\mathtt{Q}$ of our mechanical system. We will further and assume this configuration space to be embedded in a real space $\mathtt{Q} \subseteq \mathbb{R}^N$, which we will use to induce a norm on $\mathtt{Q}$. A symbol σ is expressed if the state $q(t)$ remains within distance ρ from σ for an interval of time of length δt, i.e. symbol σ was expressed at time t is equivalent to:

$$\forall s \in [t - \delta t, t] : \ \|q(s) - \sigma\| \leq \rho \tag{1}$$

We have thus obtained a definition of "(static configuration) mechanical versatility" in terms of the ability of the system to express arbitrary strings of (static configuration) symbols. From an information theoretic perspective, we thus suggest that a natural measure of mechanical versatility $\mathcal{V}$ is in bits – the number of bits needed to express the alphabet Σ, i.e. $\mathcal{V} := \log_2(\#\Sigma)$ where # is used to indicate set cardinality.

1.2. Adding mechanical realism to the system

Mechanical systems operate in the physical world, and comprise materials of limited strength, driven by power limited actuators. To capture some of this realism we add assumptions as follows. We assume our mechanism can only exert a force of F_{max} or less, because of limitations on its material properties and actuators. We assume the configuration space $\mathtt{Q}$ of our mechanism is compact. Finally, we also assume mechanism has a limited power budget W_{max}. The limited power budget and limited force together imply a limited maximal velocity V_{max} when changing configurations – regardless of the number of DOF being moved.

It should be noted that in both robotics and biology the limiting factor for actuator velocity is that actuator force decreases with speed. These limits derive from the underlying physics of the actuators themselves, and the non-zero dissipation arising from friction or fluid dynamics.

1.3. Environmental disturbances

The physical environment in which a system operates is never ideal. We will assume that the environment introduces arbitrary disturbances of bounded energy ΔE, which our system must resist.