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On the elastic plates and shells with residual surface stresses

Holm Altenbach[a,*], Victor A. Eremeyev[b,c]

[a] *Otto-von-Guericke-University Magdeburg, D-39016 Magdeburg, Germany*
[b] *Rzeszów University of Technology, Rzeszów 35959, Poland*
[c] *South Federal University, Rostov on Don 344090, Russia*

Abstract

Recently the interest grows to development of the theory of surface elasticity with respect to nanotechnologies. Nanostructured materials demonstrate very promising properties which are different from whose of bulk materials, in general. In particular, nanosized specimens exhibit size-effect that is dependence of apparent (effective) material properties such as Young's modulus on specimen's size. One of possible explanation of these phenomena is the consideration of surface-related phenomena and their influence on the effective properties at the macroscale. In fact, surface elasticity may dramatically change effective (apparent) properties of nano- and microstructure materials.

We discuss here the effective properties of thin-walled structures that are tangential and bending stiffness parameters taking into account surface/interfacial initial stresses. We consider the Gurtin-Murdoch model of surface elasticity. We show that the surface elasticity results in positive size-effect that is stiffening of nano-sized bodies in comparison with their bulk counterparts. Special attention will be paid to residual surface stresses. Unlike to surface elastic moduli there are no restrictions for values of residual stresses. In particular, we show that the compressive residual stresses may lead to negative size-effect that is to decreasing of an effective stiffness for nano-sized specimens.

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Keywords: Surface elasticity; Gurtin-Murdoch model; residual stresses; initial stresses; size-effect.

1. Introduction

The mostly used model of surface elasticity for elastic solids under large deformations was proposed by Gurtin and Murdoch [1,2]. From the physical point of view the model describes a nonlinear elastic solid with an elastic membrane attached on its surface. The stress resultant tensor acting in the membrane can be considered as a surface stress tensor. Thus, for the surface elasticity model, in addition to the three-dimensional constitutive equations, the two-dimensional constitutive relations are also required. Recently the Gurtin-Murdoch model found many applications in micro- and nanomechanics [3–6]. In particular, the surface elasticity is used for the explanation of the size-effect observed at the nanoscale. The Gurtin-Murdoch model is also applied for extension of models of plates and shells to the nanoscale,

* Corresponding author. Tel.: +49-391-671-8984; fax: +49-391-671-2863.
E-mail address: holm.altenbach@ovgu.de

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see for example [7–11], where it is shown that the surface elasticity results stiffening of nano-sized bodies in comparison with their bulk counterparts. Such behaviour is often called the positive size-effect. This result was confirmed by mathematical analysis in [12–15]. After [16] modelling of surface stresses relates also with gradient theories of elasticity, see for example [17–20].

Here we discuss the influence of residual/intial surface stresses on effective properties of thin-walled structures that is on tangential and bending stiffness parameters. Unlike to surface elastic moduli there are no restrictions for values of residual stresses. In particular, we show that the compressive residual stresses may lead to negative size-effect that is to decreasing of an effective stiffness for nano-sized specimens.

The paper is organized as follows. First we recall basic relations of the Gurtin-Murdoch model of surface elasticity [1,2] and present the linearized equations. Then within the framework of general resultant shell theory [21,22] we discuss the reduction of the considered three-dimensional boundary-value problem to two-dimensional equations of the shell theory. The through-the-thickness integration procedure is applied. Finally we discuss the influence of initial surface stress on the bending stiffness. In conclusions we briefly discuss research perspectives.

2. Basic equations of surface elasticity

The Lagrangian equilibrium equations and the boundary conditions take the following form:

$$\nabla_{\mathbf{x}} \cdot \mathbf{P} + \rho \mathbf{f} = \mathbf{0}\,, \quad (\mathbf{n} \cdot \mathbf{P} - \nabla_{\mathrm{S}} \cdot \mathbf{S})|_{\Omega_{\mathrm{S}}} = \mathbf{t}\,, \quad \mathbf{u}|_{\Omega_{\mathbf{u}}} = \mathbf{u}_0\,, \quad \mathbf{n} \cdot \mathbf{P}|_{\Omega_{\mathbf{f}}} = \mathbf{t}\,. \tag{1}$$

Here $\mathbf{P}$ is the first Piola-Kirchhoff stress tensor, $\nabla_{\mathbf{x}}$ the Lagrangian three-dimensional (3D) nabla operator, ∇_{S} the surface (2D) nabla operator, $\mathbf{S}$ the surface stress tensor of the first Piola-Kirchhoff type acting on the surfaces Ω_{S}, $\mathbf{u}$ the displacement vector, $\mathbf{f}$ and $\mathbf{t}$ the body force and surface loads vectors, respectively, and ρ the mass density. We assume that the displacements are given on the part $\Omega_{\mathbf{u}}$ of the body surface, while on $\Omega_{\mathbf{f}}$ the surface stresses $\mathbf{S}$ are absent. Equation $(1)_2$ is the so-called generalized Young-Laplace equation describing the surface tension in solids.

For a bulk material we use the standard constitutive relations of the nonlinear elasticity

$$\mathbf{P} = \frac{\partial \mathcal{W}}{\partial \mathbf{F}}, \quad \mathcal{W} = \mathcal{W}(\mathbf{F}), \quad \mathbf{F} = \nabla_{\mathbf{x}} \mathbf{x}, \tag{2}$$

where $\mathcal{W}$ is the strain energy density, $\mathbf{F}$ the deformation gradient, and $\mathbf{x}$ the position vector in the actual configuration. The surface stress tensor $\mathbf{S}$ is similar to the membrane stress resultants tensor and expressed with the use of the surface strain energy density $\mathcal{U}$

$$\mathbf{S} = \frac{\partial \mathcal{U}}{\partial \mathbf{F}_{\mathrm{S}}}, \quad \mathcal{U} = \mathcal{U}(\mathbf{F}_{\mathrm{S}}), \quad \mathbf{F}_{\mathrm{S}} = \nabla_{\mathrm{S}} \mathbf{x}\big|_{\Omega_{\mathrm{S}}}, \tag{3}$$

where $\mathbf{F}_{\mathrm{S}}$ is the surface deformation gradient.

For solids with initial/residual surface stresses we introduce the initial surface energy and initial surface stresses as follows

$$\mathcal{U}_0 = \mathcal{U}(\mathbf{F}_{\mathrm{S}}^{\circ}), \quad \mathbf{S}_0 = \frac{\partial \mathcal{U}}{\partial \mathbf{F}_{\mathrm{S}}^{\circ}}.$$

Tensor $\mathbf{F}_{\mathrm{S}}^{\circ}$ describes the initial deformation from stress-free state to the chosen initial configuration. It can be considered as the given parametric tensor in the constitutive equations. The strain energies can be considered as a functions of the right Cauchy–Green strain tensor and its surface analogues

$$\mathcal{W} = \mathcal{W}(\mathbf{C}), \quad \mathcal{U} = \mathcal{U}(\mathbf{F}_{\mathrm{S}}^{\circ} \cdot \mathbf{C}_{\mathrm{S}} \cdot \mathbf{F}_{\mathrm{S}}^{\circ\mathrm{T}}), \tag{4}$$

where $\mathbf{C} = \mathbf{F} \cdot \mathbf{F}^{\mathrm{T}}$ and $\mathbf{C}_{\mathrm{S}} = \mathbf{F}_{\mathrm{S}} \cdot \mathbf{F}_{\mathrm{S}}^{\mathrm{T}}$. For isotropic materials $\mathcal{W}$ and $\mathcal{U}$ are functions of the principal invariants

$$\mathcal{W} = \mathcal{W}(I_1, I_2, I_3), \quad \mathcal{U} = \mathcal{U}(J_1, J_2), \tag{5}$$

where $I_1 = \mathrm{tr}\mathbf{C}$, $I_2 = \frac{1}{2}\left[\,\mathrm{tr}^2\mathbf{C} - \mathrm{tr}\mathbf{C}^2\right]$, $I_3 = \det \mathbf{C}$, $J_1 = \mathrm{tr}\,\mathbf{F}_{\mathrm{S}}^{\circ} \cdot \mathbf{C}_{\mathrm{S}} \cdot \mathbf{F}_{\mathrm{S}}^{\circ\mathrm{T}}$, $J_2 = \mathrm{tr}\left(\mathbf{F}_{\mathrm{S}}^{\circ} \cdot \mathbf{C}_{\mathrm{S}} \cdot \mathbf{F}_{\mathrm{S}}^{\circ\mathrm{T}}\right)^2$.